Thermodynamic geometric analysis of D-dimensional RN black hole

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This paper also studies the thermodynamics and Ruperna geometry of the gravity of D-dimensional RN black holes. The Ruppeiner geometry of the angular momentum-fixed ensemble is curved, while the Ruppeiner geometry of the pressure-fixed ensemble is flat. We see no phase transition for the D-dimensional RN black hole(D greater than or equal to 6).

Keywords:RN black holes ,six-dimensional,D-dimensional

1. INTRODUCTION

Einstein's general theory of relativity predicted a special kind of dense celestial body that not even light could travel through. In 1783, the British geographer John-Michell proposed: If there is a celestial body with the same mass as the sun, and the radius is only 3 km, we will not be able to see this celestial body. Later, the French physicist P.S. Laplace also proposed the "dark star" theory. In 1939, H. Snyder and J.R. Oppenheimer calculated through general relativity that when the mass of a neutron star is greater than the critical value, exploration will occur and a black hole will be formed. In 1968, John Wheeler first named such scientifically predicted celestial bodies "black holes".

As early as 1916, gravitational waves were theoretically proved by Einstein, and the laser principle was also proposed, which provided the necessary theoretical basis for the development of subsequent laser transmitters, and based on laser technology, humans realized the detection of gravitational waves, On February 11, 2016, the LIGO team announced the first detection of gravitational waves. The first human black hole photo was also released in 2019.

According to the classical black hole theory, when a star collapses in space-time, all information such as mass will be lost, so researchers propose the black hole hairless theorem. Therefore, based on the different parameters of black holes, classical black holes can be divided into four forms, namely: (1) rotating charged black holes; (2) rotating uncharged black holes; (3) non-rotating charged black holes; (4) non-rotating and uncharged black holes black hole. The above four types of black holes correspond to Kerr-Newman metric; Kerr metric; R-N metric; Schwarzschild metric. After the concept of black hole was put forward, people began to discuss how to determine the boundary of black hole. But until today, there is still no perfect solution for the definition of the black hole boundary. In the vicinity of the black hole, there is a critical point that cannot be returned, namely the event horizon.

The study of wave and particle absorption by black holes and their higher-dimensional objects has received increasing attention over the past 50 years, as many fundamental aspects of classical and quantum black hole physics are directly related to this topic, and we It can also better understand the properties of space and time. Research in this field began in the 1970s with the study of linear perturbations having a long history in general relativity (GR)[1–9]. Regge and Wheeler[6] pioneered the perturbation analysis of the space-time of black holes (BHs) and demonstrated the importance of this analysis in many fields from astrophysics to high-energy physics. There are two possible outcomes when the black hole is subjected to tiny vibrations: the perturbed black hole is stable, which is due to the effect of the external damping mechanism of the black hole; when the initial conditions are destroyed, the unconstrained black hole is unstable under the perturbation, which will Inevitably disappears or evolves into another stable object. While astrophysicists believe that black holes are stable, with small fluctuations, many solutions to the black hole problem may be undermined by new instabilities. Perturbation theory reveals that black holes vibrate in a nice way, exhibiting a discrete spectrum of better vibrational modes known as the positive-like scale. Linear perturbation theory has been an active area of research for decades and has proven to be a very practical tool in analytical and numerical calculations for testing the modal stability of black holes and dense objects[1–5]. Various studies have revealed that spacetime becomes unstable under perturbation.

Ruppeiner geometry is based on waves. When the microstructure of the thermodynamic system under study is unknown, Ruperna geometry provides us with a powerful tool for exploring the microstructure of the thermodynamic system. The measure of Ruppeiner geometry is[10]

$$ds^2 = -\frac{\partial^2 S}{\partial X^\alpha \partial X^\beta}\Delta X^\alpha \Delta X^\beta \tag{1}$$

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where S is the entropy of the thermodynamic system, $\Delta X^{\alpha} = X^{\alpha} - X_0^{\alpha}$ is the thermodynamic quantity X^{α} deviates from equilibrium The fluctuation of the thermodynamic quantity X_0^{α}. The physical interpretation of this metric is clear: the farther away you are from equilibrium in phase space, the less likely you are to be at that point. For ordinary thermodynamic systems, if the thermodynamic system under study is fermions, the curvature scalar of Ruperna geometry is positive; if the thermodynamic system under study is bosons, the curvature scalar of Rupera geometry is negative; for ideal classical gases , the curvature of the Ruppeiner geometry is scalar zero. Further research shows that if the curvature scalar is positive, the microscopic interaction of the system is repulsive; if the curvature scalar is negative, the microscopic interaction of the system is attractive.

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2. DESCRIPTION OF THE SYSTEM

We first propose our model with a non-extremal D-dimensional RN black hole, and then take the extreme limit for further discussion. The measure of a D-dimensional non-extremal RN black hole is[11]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{D-2}^2 \tag{2}$$

The function $f(r)$ reads

$$f(r) = 1 - \frac{2m}{r^{D-3}} + \frac{q^2}{r^{2(D-3)}}, \tag{3}$$

where the parameters m and q are related to the ADM mass M and charge Q of the RN black hole,

$$m = \frac{8\pi}{(D-2)\operatorname{Vol}\left(S^{D-2}\right)} M, \quad q = \frac{8\pi}{\sqrt{2(D-2)(D-3)}\operatorname{Vol}\left(S^{D-2}\right)} Q. \tag{4}$$

Here $\operatorname{Vol}\left(S^{D-2}\right) = 2\pi^{\frac{D-1}{2}}/\Gamma\left(\frac{D-1}{2}\right)$ is the volume of the unit (D-2)-sphere. $d\Omega_{D-2}^2$ is the common line element of the (D-2) dimensional unit sphere S^{D-2}, which can be written as

$$d\Omega_{D-2}^2 = d\theta_{D-2}^2 + \sum_{i=1}^{D-3} \prod_{j=i+1}^{D-2} \sin^2\left(\theta_j\right) d\theta_i^2 \tag{5}$$

The range of angular coordinates is taken as $\theta_i \in [0, \pi](i = 2, .., D-2), \theta_1 \in [0, 2\pi]$. The inner and outer horizons of this RN black hole are

$$r_{\pm} = \left(m \pm \sqrt{m^2 - q^2}\right)^{1/(D-3)}. \tag{6}$$

Obviously we have the following two equations

$$r_+^{D-3} + r_-^{D-3} = 2m, r_+^{D-3} r_-^{D-3} = q^2. \tag{7}$$

3. THERMODYNAMIC PARAMETERS OF D-DIMENSIONAL RN BLACK HOLE

We list a special case of six-dimensional RN black holes, and it is easy to deduce that the thermodynamic geometric phase transition properties of six-dimensional black holes are isomorphic with those of D-dimensional RN black holes:The metric of $D = 6$RN black hole is[14]

$$ds_6^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_4^2 \tag{8}$$

where

$$f(r) = 1 - \frac{2m}{r^3} + \frac{q^2}{r^6} \tag{9}$$

The parameters $m1$ and q are respectively related with the ADM mass M and the electric charge $Q1$ of the black hole,

$$m1 = \frac{3}{4\pi} M, q = \frac{3}{2\sqrt{6}\pi} Q1 \tag{10}$$

The inner and outer horizons of the black hole are

$$r_{\pm} = \left(m \pm \sqrt{m^2 - q^2}\right)^{1/3}. \tag{11}$$

The six-dimensional RN black holes temperature from the Hawking-Bekenstein relationship $T = f'(r_+)/4\pi$ will be (in the following relationship, we have introduced their horizon for events and quality parameters):

$$T = f'(r_+)/4\pi = (-\frac{6q^2}{r_+^7} + \frac{6m}{r_+^4})/4\pi \tag{12}$$

In addition, the entropy of this six-dimensional RN black holes solution is

$$S \to {r_+}^4 \tag{13}$$

The metric of Ruppeiner geometry is

$$ds^2 = -\frac{\partial^2 S(m,q)}{\partial X^\alpha \partial X^\beta} \Delta X^\alpha \Delta X^\beta \tag{14}$$

$$g_{ij} = \begin{pmatrix} \frac{4\left(2m^3 - 5mq^2 + 2m^2\sqrt{m^2-q^2} - 4q^2\sqrt{m^2-q^2}\right)}{9(m^2-q^2)^{3/2}\left(m+\sqrt{m^2-q^2}\right)^{2/3}} & \frac{4q\left(2m^2+q^2+2m\sqrt{m^2-q^2}\right)}{9(m^2-q^2)^{3/2}\left(m+\sqrt{m^2-q^2}\right)^{2/3}} \\ \frac{4q\left(2m^2+q^2+2m\sqrt{m^2-q^2}\right)}{9(m^2-q^2)^{3/2}\left(m+\sqrt{m^2-q^2}\right)^{2/3}} & -\frac{4\left(3m^3+3m^2\sqrt{m^2-q^2} - q^2\sqrt{m^2-q^2}\right)}{9(m^2-q^2)^{3/2}\left(m+\sqrt{m^2-q^2}\right)^{2/3}} \end{pmatrix} \tag{15}$$

The curvature scalar of thermodynamic geometry R(S) is

$$R(S) = 0. \tag{16}$$

The thermodynamic geometric curvature scalar of the four-dimensional RN black hole is compared as[15, 16]

$$\hat{R} = g_{ab}R^{ab} = -\frac{r_+ - r_-}{\pi r_+ \left(3r_- - r_+\right)^2}. \tag{17}$$

We see that when the RN black hole is an extreme black hole $r_+ = r_-$, the curvature scalar vanishes equal to zero. In general, the curvature scalar is not equal to zero. When $r_+ = 3r_-$, the curvature scalar is equal to negative infinity. According to Ruppeiner's theory[15, 16], it represents a phase transition of the system.

4. SUMMARY

This paper also studies the thermodynamics and Ruperna geometry of the gravity of D-dimensional RN black holes. The Ruppeiner geometry of the angular momentum-fixed ensemble is curved, while the Ruppeiner geometry of the pressure-fixed ensemble is flat. We see no phase transition for the D-dimensional RN black hole(D greater than or equal to 6).

By discussing the thermodynamic geometric properties of RN black holes in D-dimensional RN (D greater than or equal to 6), we find some interesting conclusions, some of which are different from others. In our detailed discussion, we compare the black hole and the van der Waals-Maxwell system when we choose Ruppeiner geometry as the thermodynamic potential, and use the corresponding relationship to select ϕ as the parameter, instead of choosing an extended quantity as in the past. In this way, we obtain the curvature scalars of the two geometries, and there is a clear difference between the RN black hole and the D-dimensional RN (D greater than or equal to 6) RN black hole.

Conflicts of Interest: The authors declare no conflict of interest.

Author Contribution Notes

According to the examples of author contribution expressions in international mainstream authoritative journals, author contributions can be divided into the following:

(1) The proposal and design of research propositions, including the proposal of a specific viewpoint or method;
(2) the conduct of research processes, such as conducting experiments or surveys;
(3) Acquisition, provision and analysis of data;
(4) Drafting of the thesis or revision of the final version.

Each research paper may describe author contributions in detail according to the characteristics of its research activity. Some research activities may also involve other research work and contributions, which can be further specified. A single author may be responsible for many aspects of the content, and multiple authors may be responsible for the same content.

Wen-Xiang Chen: As the first author, the main work - the conduct of research processes, such as conducting experiments or surveys;Acquisition, provision and analysis of data;Drafting of the thesis or revision of the final version.

Yao-Guang Zheng : As the corresponding author, his job is that the proposal and design of research propositions, including the proposal of a specific viewpoint or method.

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[1] Matzner, Richard A. Scattering of Massless Scalar Waves by a Schwarzschild "Singularity"[J]. Journal of Mathematical Physics, 1968, $9(1):163-170$.

[2] Mashhoon, Bahram. Scattering of Electromagnetic Radiation from a Black Hole[J]. Physical Review D, 1973, 7(10):2807-2814.

[3] Starobinskii A A. Amplification of waves during reflection from a rotating "black hole"[J]. Journal of Experimental and Theoretical Physics, 1973 , $37(1):28$.

[4] Starobinskii A A, Churilov S M. Amplification of electromagnetic and gravitational waves scattered by a rotating[J]. Zhurnal Eksperimentalnoi I Teroreticheskoi Fiziki, 1974, 38(1): 1 .

[5] Fabbri R. Scattering and absorption of electromagnetic waves by a Schwarzschild black hole[J]. Phys. Rev. D, 1975, 12(4):933-.

[6] Regge T, Wheeler J A. Stability of a Schwarzschild Singularity[J]. Physical Review, $1957, 108(4):1063-1069$.

[7] V. Khatsymovsky, Phys. Lett. B 320, 234-240 (1994).

[8] M. Visser, Lorentzian Wormholes: From Einstein to Hawking (American Institute of Physics, 1995), ISBN 978-1-56396-653-8.

[9] A. Einstein and N. Rosen, Phys. Rev. 48, 73 (1935).

[10] Chen, Wen-Xiang, Yao-Guang Zheng, and Jing-Yi Zhang. "Thermodynamic geometric analysis of Schwarzschild and RN black hole under f (R) gravity." arXiv preprint arXiv:2112.06718 (2021).

[11] R. C. Myers and M. J. Perry, Annals Phys. $172, 304$(1986).

[12] Hempfling, Ralf. "Yukawa coupling unification with supersymmetric threshold corrections." Physical Review D 49.11 (1994): 6168.

[13] Zubair, M., Saira Waheed, and Yasir Ahmad. "Static spherically symmetric wormholes in f (R, T) gravity." The European Physical Journal C 76.8 (2016): 1-13.

[14] R. A. Konoplya and A. Zhidenko, Phys. Rev. Lett. 103, 161101 (2009).

[15] Aman J. E., Bengtsson I., Pidokrajt N., Geometry of Black Hole Thermodynamics, General Relativity and Gravitation, 35, 2003, 1733-1743.

[16] Aman J. E., Pidokrajt N., Geometry of higher-dimensional black hole thermodynamics, Physical Review D, 73, 2006, 024017.