Snell's law of reflection from Apollonius circle

Radhalrishnamurty Padyala
Retired Scientist, Central Electrochemical Research Institute, Karaikudi
282, DMLO, Yelahanka, Bangalore 560064, India
Email: padyala1941@yahoo,com
ORCID ID: 0000-0001-6029-6155

Abstract

We show in this note that the definition of Apollonius circle is an alternate form of the statement of Snell's law of reflection. Using the definition of Apollonius circle we construct the path of a light ray between two given points reflected at a point on the circle in traversing the path from one given point to the other given point.

Keywords:

Snell's laws, Reflection, Apollonius circle, Spherical surface, Orthogonal circles

Introduction

The phenomenon of reflection of light is well known since antiquity[1-3]. It is even employed in practical applications, the most important among them being warfare. Sun's rays could be concentrated to a unique point using concave spherical mirrors, raising the temperature at that point to very high values enough to burn even ships at sea. Archimedes is said to have used it[3]. The law governing the phenomenon is known from antiquity. Euclid demonstrated that the incident ray and the reflected ray make equal angles with the surface of reflection. In due course of time, the law is stated in terms of the equal angles made with respect to the normal to the surface of reflection at the point of incidence. Both are equivalent. While the path of reflection at plane surfaces appeals to common sense, reflection at curved surfaces is not so. In this context reflection of light at concave spherical surfaces assumes special significance. Apollonius of Perga developed a geometrical theory of conics which helps in understanding reflection of light at concave spherical surfaces. In his theory the definitions of conics are different from the ones used today, especially using algebraic equations. In this note we show that Snell's law of reflection of light follows from the definition of Apollonius circle. In our analysis we use the 2D version (the planar) of the 3D spherical reflectors.

Apollonius definition of a circle[4]

'If a point moves in such a way that its distance from one fixed point is always a constant multiple of its distance from another fixed point, then its path is a circle.' It is the Apollonius circle.

Let A, B be two given fixed points. Let P be a movable point. P can be anywhere provided that

$$AP = k.BP \qquad (1)$$

Where the constant k, is any positive number. We take $k > 1$, in the following. (Similar arguments hold for $k < 1$). If the line segment is divided by points C, D harmonically such that,

$$AC : CB = AD : BD = k : 1 \qquad (2)$$

Then the circle described with CD as diameter is the Apollonius circle (Fig. 1). P lies on it.

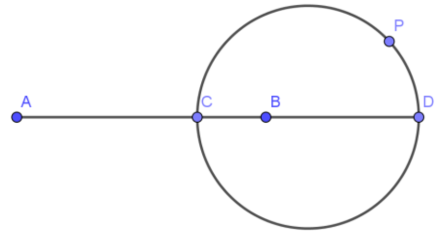

Fig. 1 A, B are two given fixed points, The line segment AB is divided harmonically by points C and D in the ratio k : 1. The circle with CD as diameter is the Apollonius circle.

For different values of k, we get different circles which constitute a family of coaxial non intersecting circles. Let us call it the family of α circles. For k < 1 we get the circles to the left of the perpendicular bisector of AB.

The point P on this Apollonius circle satisfies the relation

$$AC : CB = AP : BP = AD : BD = k : 1 \qquad (3)$$

Join A, P; B , P; C , P; B , P (Fig. 2). In the triangle APB the ratio of the sides AP and BP is equal to the ratio of the components AC and CB of AB. Therefore, angle APB is bisected by the line PC. In other words, PC is the bisector of angle APB.

Snell’s law of reflection[1]

Snell’s law of reflection states that the angle of incidence is equal to the angle of reflection. The incident ray, the reflected ray and the normal to the surface of reflection at the point of incidence, all lie in the same plane. Snell’s law of reflection demands precisely the conditions above.

If AP is a ray of light from A incident on the Apollonian circle at point P, then the ray is reflected to pass through B along PB. This is because the rays AP and BP make equal angles with chord CP, which is the normal at P to the reflecting surface of the circle represented by the chord DP (see Fig. 2). Angle APC is the angle of incidence and Angle CPB is the angle of reflection. These two angles are equal since angle APB is bisected by the line PC.

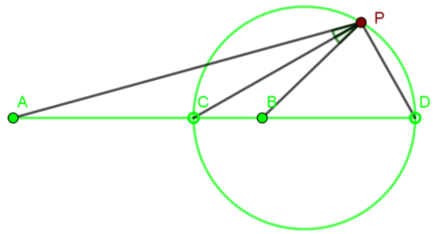

Fig. 2 Reflection of a light ray at a point on Apollonius circle.

Draw the circle passing through A, B, P. The circle is shown in red color in Fig. 3. It intersects the Apollonian circle shown in green color, orthogonally at P. That means, the tangents drawn to the two circles at P are perpendicular to each other. For different locations of P we get different (red) circles T he group of these circles constitutes a family of coaxial intersecting circles. We call it the family of β circles. Every member of the β family intersects every member of the α family of circles orthogonally. The centers of all the β circles lie on the perpendicular bisector of AB. It is called the radical axis. For more details see Ogilvy, (ref. 4).

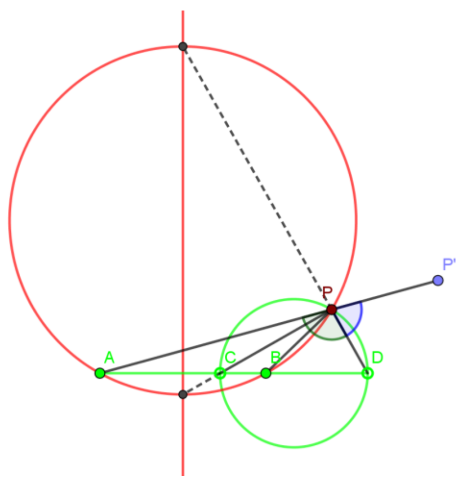

Fig. 3 Figure shows the movable point P on the Apollonian circle. The circle passing through points A, B, P is shown in red color. It is a member of the family of β circles.

It is important to note that it is not the tangent plane to the circle which acts as the surface of reflection, but the plane perpendicular to CP that acts as the surface of reflection (Fig. 3).

Thus every point P on the Apollonius circle reflects rays of light coming from one of the given points say A, to pass through the other given point B. For each location of P we get a different β circle that intersects the α circle at the new location of P (see Fig. 4).

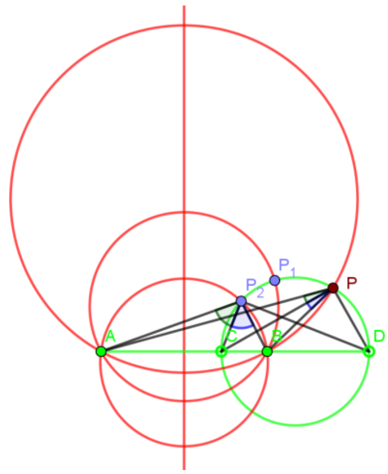

Fig. 4 Figure shows the β circles (red) intersecting the Apollonius circle (green) orthogonally at different locations of P on it. Light rays from A are reflected at Ps to pass through B.

While PC bisects the angle APB internally, PD bisects the angle APB externally (see Fig. 5).

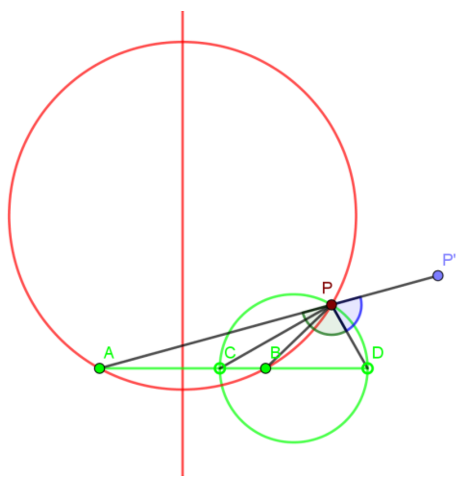

Fig. 5. Figure shows that PD is the external bisector of the angle APC.

Snell's law of reflection immediately follows for both internal and external bisection of the angle as is clear from the following equations.

$$\frac{\sin\,(A\hat{P}C)}{\sin\,(B\hat{P}C)} = \frac{\sin\,(A\hat{P}D)}{\sin\,(D\hat{P}P')} = \frac{\sin\,(A\hat{P}D)}{\sin\,(180 - A\hat{P}D)} = \frac{\sin\,(A\hat{P}D)}{\sin\,(A\hat{P}D)} = 1 \qquad (4)$$

Since (4) holds for any location of P on Apollonius circle, it follows that the definition of Apollonius circle is an alternate form of the statement of Snell's law of reflection.

Acknowledgment

It is a pleasure to thank Mr. Arun Rajaram, Chennai, for the unstinted support and encouragement he gives for my research work.

References

1. A. M. Smith, Ptolemy and the Foundations of Ancient Mathematical Optics: A Source Based Guided Study, Trans. Am. Phil. Soc., New Series, **89**, N0. 3, (1999). https://www.jstor.org/stable/3185870.
2. Pavlos Mihas, Some historical resources for teaching about curved mirrors, Phys. Educ. **37**, (2002) 311.
3. Wilbur Knorr, The Geometry of Burning - Mirrors in Antiquity, ISIS,***74***, (1983) 53-73.
4. C. Stanley Ogilvy. *Excursions in geometry*, Dover Pub. Inc. New York, (1990) Ch. 2, pp. 13-23.

Statement of Conflict of interest (COI) Disclosure

There is no conflict of interest. I did not receive any funds from any source for this work.