The velocity center theorem of a spinning mobile object and its application to the vehicles
Application to the automobiles

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Abstract

In the previous report ”The velocity center theorem of a spinning mobile object and its application to the vehicles” (DOI: https://doi.org/10.51094/jxiv.314) , described the application of the velocity center theorem to the mobility. In this report, consider the case of applying the velocity center theorem to the front wheel steering automobiles. When the automobile moves forward while changing its direction, the velocity center point CV is on the wheel axial intersection point, so CV is on the extension line of rear wheel axis.,

When the traveling direction of the representative point CS and the direction of the automobile' center line do not match, and the point CS goes straight, the center line of the automobile gradually approaches the traveling direction of point CS.

If the point CS goes curve line, the turning center CT of CS is on the straight line through CS and CV.

And when point CV and point CT do not match, point CV gradually approaches point CT, so the whole of the automobile approaches to the turning motion centered on point CT. Also, when r_{CS} is defined as the turning radius of CS that is the distance from CS to CT, θ is the angle of automobile's center line from the traveling direction of CS, L is the distance from rear wheel axis to point CS, and δ is defined as $\arcsin(\mathrm{L}/r_{CS})$, θ converges to $-\delta$. (If CS goes straight, $\delta = 0$)

The wheel axial intersection point (CV) is often referred to as the center of turning motion, but it is not necessarily that the turning center point CT is on the extension lines of wheels axis.

These show that the representative point CS is able to correct its posture in the proper direction while moving along the ideal trajectory.

This method is useful for autonomous driving and posture control etc.

Keywords : automobile、The velocity center theorem、autonomous driving、driving stability、posture control

1. Introduction

In the current theory of the automobile, it can't be found an explanation that corrects posture while moving the ideal orbit. In this report, the velocity center theorem of the previous report ” The velocity center theorem of a spinning mobile object and its application to the vehicles”(DOI: https://doi.org/10.51094/jxiv.314) is applied to the automobile, and if there is a GAP in the direction of automobile body against the ideal orbit, it shows a representative example that corrects posture while moving the ideal orbit.

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2. The relationship between the velocity center theorem and the automobiles

2.1 The velocity center theorem of a spinning mobile object and its application to the automobiles

The velocity center theorem stated in the previous report is as follows,
On the same plane,
when the representative point CS on an object U moves velocity $\overrightarrow{V_{CS}}$ with spinning angular velocity Ω_{cs},
The point CV is the position of the distance $|\overrightarrow{V_{CS}}|/|\Omega_{cs}|$ from point CS,
And its direction is the direction from $\overrightarrow{V_{CS}}$ to $\pi/2$ rad (When $\Omega_{cs} > 0$) or $-\pi/2$ rad (When $\Omega_{cs} < 0$).

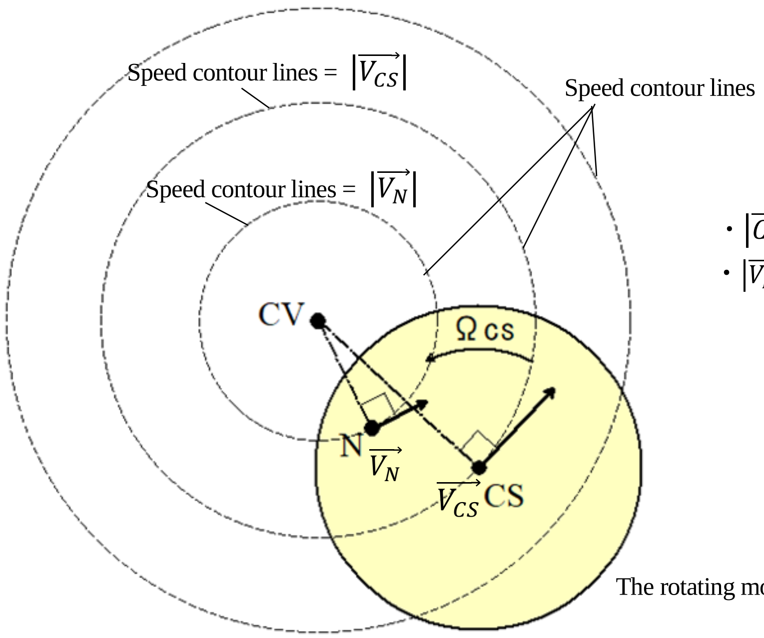
The absolute velocity $\overrightarrow{V_N}$ at any point N on the rigid object U is expressed as follows,

$$\overrightarrow{V_N} = |\Omega_{cs}|\ Rot\left(\frac{\pi}{2} - \alpha\right)\overrightarrow{CV\text{-}N} = \Omega_{cs}\ Rot\left(\frac{\pi}{2}\right)\overrightarrow{CV\text{-}N}$$

At this time、α、$Rot(\theta)$ and other condition are as follows.

- Point CS on the object U, mass point N, and velocity $\overrightarrow{V_{CS}}$ of point CS are coplanar (on the same plane S).
- $Rot(\theta)$ is the rotation matrix that rotate θrad on the same plane S
- α is a value that satisfies the following conditions.
  When $\Omega_{cs} > 0$, $\alpha = 0$.
  When $\Omega_{cs} < 0$, $\alpha = -\pi$.
- The point CV is the position of the distance $|\overrightarrow{V_{CS}}|/|\Omega_{cs}|$ from point CS,
  And the direction of $\overrightarrow{CS\text{-}CV}$ is the direction from $\overrightarrow{V_{CS}}$ to $\pi/2+\alpha$ rad
- $\overrightarrow{CV\text{-}N}$ is the vector from the point CV to the point N.

Therefore, the speed $|\overrightarrow{V_N}|$ is $|\Omega_{cs}||\overrightarrow{CV\text{-}N}|$,
And the direction of $\overrightarrow{V_N}$ is the direction from $\overrightarrow{CV\text{-}N}$ to $\pi/2$ rad (When $\Omega_{cs} > 0$) or $-\pi/2$ rad (When $\Omega_{cs} < 0$).
The image diagram is like Figure 2.1-1.



- $|\overrightarrow{CS\text{-}CV}| = |\overrightarrow{V_{CS}}|/|\Omega_{CS}|$
- $|\overrightarrow{V_N}| = |\Omega_{CS}||\overrightarrow{CV\text{-}N}|$
  $= |\overrightarrow{V_{CS}}||\overrightarrow{CV\text{-}N}|/|\overrightarrow{CS\text{-}CV}|$

Fig. 2.1-1 The relationships between CS, CV and N of the rotating mobile rigid object

When the velocity center theorem is applied to the front wheel steering car, for example, it looks like Figure2.1-2.

The point CV is on the rear wheel axis extension line. And if the point CS is not on the rear wheel axis and its relative position from the rear wheel axis does not change, CS can be set anywhere, whether inside or outside of the automobile, such as the center of gravity, driver's seat, outline or outside of the car body, etc. Where to set CS is determined by the situation.

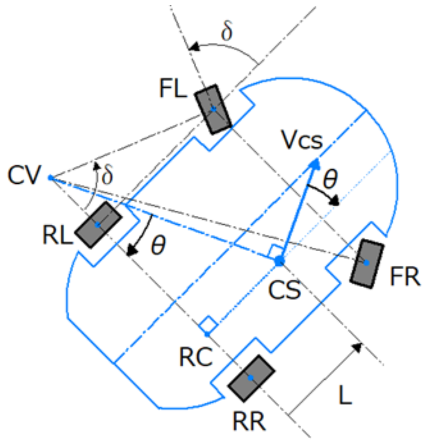


Fig. 2.1-2 Relationships between CS,CV and a front wheel steering automobile

Let θ be the angle that the automobile center line from the reference line CS traveling direction.
At this time, θ is equal to the angle of the rear wheel axis from the reference line CV-CS.
Also in Fig.2.1-2, θ is defined as positive when counterclockwise and negative when clockwise.
If CS is at distance L from the rear wheel axis, and the direction from the rear wheel axis to the front is defined as positive
The distance $s_{\overline{cs\text{-}cv}}$ from CS to CV is expressed as next.

$$s_{\overline{cs\text{-}cv}} = -\frac{L}{sin\theta}$$

If facing the traveling direction from CS, $s_{\overline{cs\text{-}cv}}$ takes a positive value when CV is on the left side, and a negative value when on the right side.
Also, from the velocity center theorem,

$$V_{CS} = s_{\overline{cs\text{-}cv}}\, \Omega_{CS},$$

so Ω_{CS} is expressed as next.

$$\Omega_{CS} = -\frac{sin\theta}{L} V_{CS}$$

The speed of each wheel is equal to the multiplication of Ω_{cs} and the distance from CV to each wheel,
And each wheel-steer-angle is equal to the angle of the straight line through CV and each wheel from the reference line rear wheel axis.
In fact, there are restrictions on the wheel angle, so when the steering wheel is operated to the limit, the absolute value of distance $s_{\overline{cs\text{-}cv}}$ from CS to CV is the smallest. With that minimum value as $|s_{\overline{cs\text{-}cv}}|_{min}$, for $s_{\overline{cs\text{-}cv}} = -L/sin\theta$,
the range of controllable θ is the range that satisfies the following.

$$|sin\theta| \leq \left|\frac{L}{|s_{\overline{cs\text{-}cv}}|_{min}}\right| \qquad \left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$$

So, the smaller L, the narrower the θ controllable range.
If there is no wheel angle limit, there is no θ limit.

2.2 Posture correction of linear motion

In this chapter, describes the case that CS moves on a straight line. As with previous chapter, set θ like Fig.2.2-1(A), and set CV, the direction and speed of wheel to satisfy the velocity center theorem. So, the absolute value of θ changes as it approaches 0 like (B), and finally, approaches the position of (C).

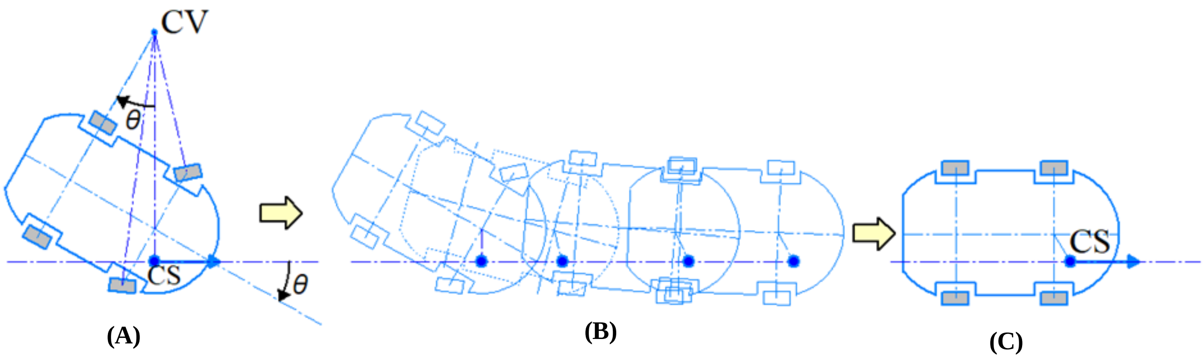


Fig.2.2-1 Image of automobile's movement when point CS goes straight

If this movement is considered in the velocity center theorem's formula, it is as follows.

This paper uses the right-hand coordinate system - the x axis points to the right, the y axis points up, and the counterclockwise direction is positive(the z axis points out of this paper), as seen in the above Fig.2.2-1.

When the velocity center theorem is represented by coordinates, it is as follows.

$$\overrightarrow{V_{CS}} = \Omega_{CS}\, Rot\left(\frac{\pi}{2}\right) \overrightarrow{CV\text{-}CS}$$

$$\begin{pmatrix} V_{CS} \\ 0 \end{pmatrix} = \frac{d\theta}{dt}\, Rot\left(\frac{\pi}{2}\right) \begin{pmatrix} 0 \\ \dfrac{L}{sin\theta} \end{pmatrix} = \frac{d\theta}{dt} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \dfrac{L}{sin\theta} \end{pmatrix} = \frac{d\theta}{dt} \begin{pmatrix} \dfrac{-L}{sin\theta} \\ 0 \end{pmatrix}$$

Therefore, the next equation will hold.

$$V_{CS} = -\frac{L}{sin\theta}\frac{d\theta}{dt}$$

So, it can be transformed as follows.

$$\int \frac{V_{CS}}{L} \mathrm{dt} = -\int \frac{d\theta}{sin\theta}$$

$$cos\theta = 1 - \frac{2}{e^{2\beta} e^{2\int \frac{V_{CS}}{L} dt} + 1}$$

Here, $e^{2\beta}$ (>0) is a constant value that determined by the initial value.

When L is a constant value, and defines $U(= \int V_{CS} dt)$ as the distance that CS goes,

And define dimensionless quantity $q = U/L$, above equation is expressed as the following equation.

$$cos\theta = 1 - \frac{2}{e^{2(q+\beta)} + 1} \qquad (※1)$$

Fig.2.2-2 is a graph of next equation ※2.

$$cos\theta = 1 - \frac{2}{e^{2q} + 1} \qquad (※2)$$

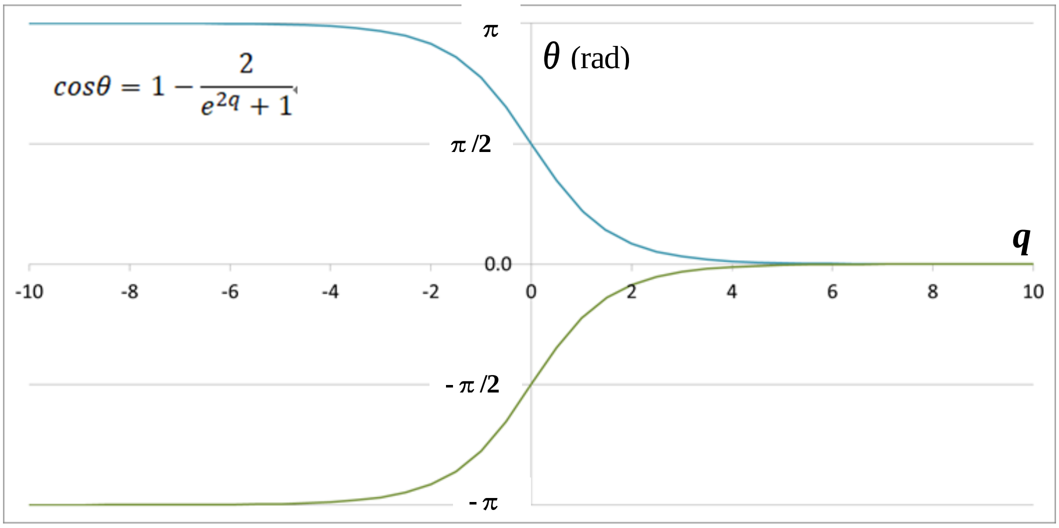


Fig.2.2-2 Relationships between q and θ , when the point CS goes straight

The graph of equation ※1 is a graph with only moved –β in the q-axis direction from the graph of equation ※2.
In this reason, if q at the time of θ_1, θ_2, are q_1, q_2 respectively,
the amount of change $\Delta q = q_2 - q_1$ that required to become θ_1 to θ_2 is a value that is not affected by β .
The ***q*** in Fig.2.2-2 can be replaced by the distance axis U (=qL) and by the time axis T (=qL/*Vcs*) when *Vcs* is constant.
The smaller L is, the shorter U and T required to converge to the target, but there is also the contradictory disadvantage of narrowing the range of controllable θ as shown in Chapter 2.1.
It is needed to choose the right L according to the situation.
Also, Erase θ from $s_{\overline{cs\text{-}cv}} = -L/sin\theta$ and the equation ※2, and define a dimensionless quantity $p = s_{\overline{cs\text{-}cv}}/L$,
a relationship between p and q can be represented simple as follows.

$$p = \pm\frac{1}{2}(e^{q} + e^{-q}) \quad (\mathrm{L} > 0, \text{and when } \theta < 0, p \text{ is "+"}, \text{and when } \theta > 0, \text{" } - \text{ "})$$

The specific examples so far are shown next. For example, In the formula ※2,
If θ changes fromπ/6 rad (30°) toπ/1800 rad(0.1°),
$q_{\pi/6} \fallingdotseq 1.317$ when θ=π/6 rad , and $q_{\pi/1800} \fallingdotseq 7.044$ when θ=π/1800 rad,
So, the amount of change q required to become θ=π/6 rad to θ=π/1800 rad is represented as next,
$\Delta q = q_{\pi/1800} - q_{\pi/6} = 5.727$
For example, if L = 2m, *Vcs* =10m/s,
The distance required to reach the target is $\Delta U = \Delta q\ L = 11.454\text{m}$, the time is $\Delta T = \Delta \mathrm{U}/V_{CS} = 1.1454\text{sec}$.
However, the automobile satisfies $|s_{\overline{cs\text{-}cv}}|_{min} \leq 2/0.5 = 4m \leq |L/sin\theta|$ is necessary.
If it is not satisfy the conditions , L and V_{CS} must be adjusted to meet the conditions.

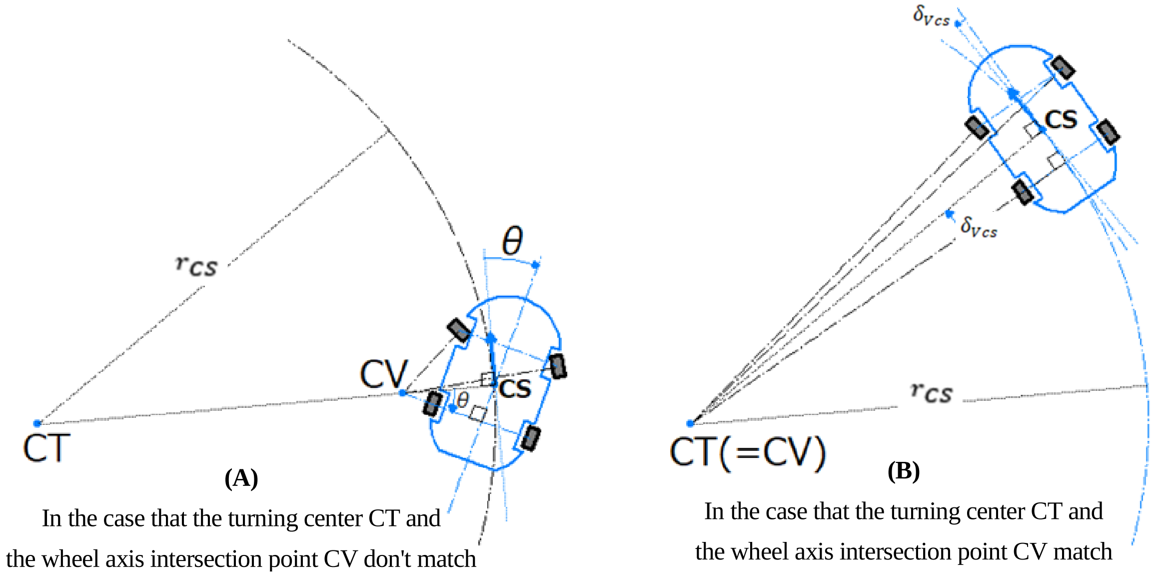
2.3 Posture correction of turning motion on the arc orbital line

In this chapter, like Fig.2.3-1, describes the case that CS makes a turning motion on the arc orbit of radius r_{CS}(curvature $\rho_{CS} = 1/r_{CS}$) centered on point CT. Additionally as in the previous chapter, set θ, CV, L etc., those satisfy the velocity center theorem.

And define δ_{Vcs} $(-\frac{\pi}{2} \leq \delta_{Vcs} \leq \frac{\pi}{2})$ that satisfies $sin\delta_{Vcs} = \rho_{CS}L = \frac{L}{r_{CS}}$.

δ_{Vcs} is a positive value when CS goes counterclockwise direction and a negative value when clockwise

When CT and CV do not match, like Fig.2.3-1(A) and if CS goes on an arc orbital line while satisfying the velocity center theorem, θ goes closer to $-\delta_{Vcs}$, and CV goes closer to CT. Then, gradually approach the state of Fig.2.3-1(B), like the whole car body is in a state of turning arc motion centered on the point CT(=CV).



(A)
In the case that the turning center CT and the wheel axis intersection point CV don't match

(B)
In the case that the turning center CT and the wheel axis intersection point CV match

Fig. 2.3-1 The image of a car's turning motion when CS moves on an arc orbit centered on CT

A typical graph of θ and q of this turning motion corresponding to Fig.2.2-1 is expressed like next Fig.2.3-2.

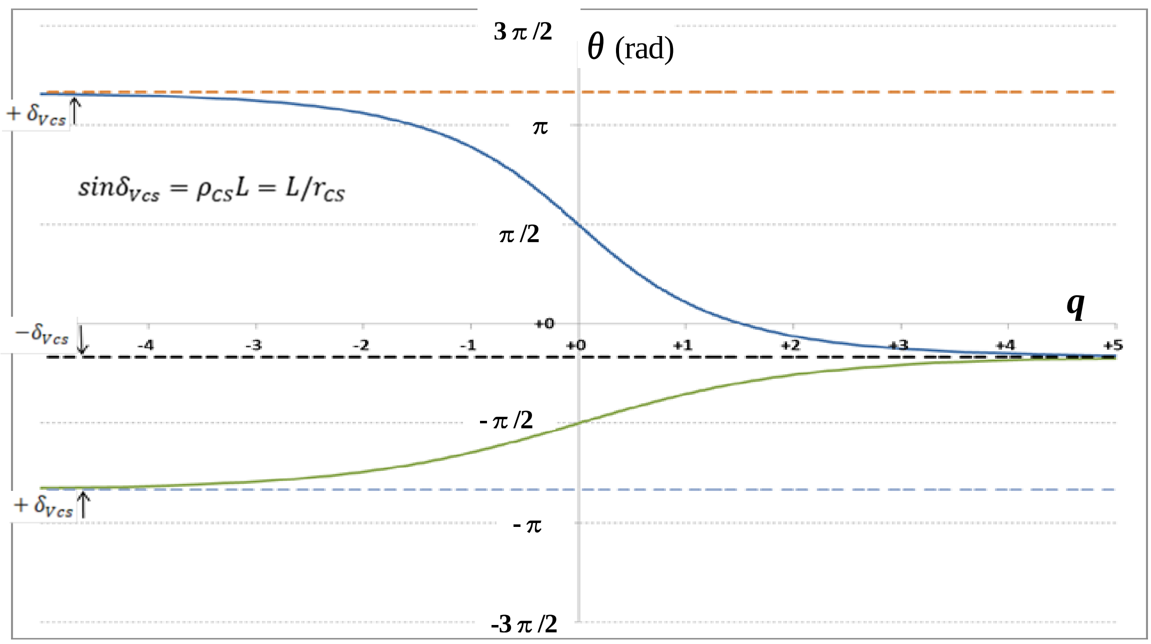


Fig.2.3-2 Relationships between q and θ, when CS moves on the arc orbit centered on CT

$\theta = -\delta_{Vcs}$ as the boundary, the curve of Fig.2.3-2 is divided into the case of $-\pi + \delta_{Vcs} \le \theta < -\delta_{Vcs}$ and the case of $-\delta_{Vcs} < \theta \le \pi + \delta_{Vcs}$, and both curves, as q increases, θ converges to $-\delta_{Vcs}$.
When there is curvature in the orbit, the rate of convergence differs on the positive side and negative side of the convergence value $\theta = -\delta_{Vcs}$.
This shows that the rate of change in C to correct the posture differs from the inside and the outside of the orbit.
When $\delta_{Vcs} = 0$ (straight line), like the chapter 2.2, the curve becomes line-symmetrical against the q-axis.

Fig.2.3-2 shows an example of the case of $\delta_{Vcs} > 0$ and when (q , θ) = (0 , ± π / 2).
When expressed in the formula, it is as follows.

When $(sin\theta + sin\delta_{Vcs})sin\delta_{Vcs} > 0$ and $\delta_{Vcs} > 0$,
$-\delta_{Vcs} < \theta < \pi + \delta_{Vcs}$

$$\theta = 2\tan^{-1}\left\{\frac{1}{sin(\delta_{Vcs})}\left\{\frac{1 + exp[-(q-b)cos(\delta_{Vcs})]}{1 - exp[-(q-b)cos(\delta_{Vcs})]}\ cos(\delta_{Vcs}) - 1\right\}\right\}$$

$$b = \frac{1}{cos(\delta_{Vcs})}\, ln\left|\frac{sin(\delta_{Vcs}) + 1 - cos(\delta_{Vcs})}{sin(\delta_{Vcs}) + 1 + cos(\delta_{Vcs})}\right| \qquad \left(\text{when } q = 0,\ \theta = \frac{\pi}{2}\right)$$

When $(sin\theta + sin\delta_{Vcs})sin\delta_{Vcs} < 0$ and $\delta_{Vcs} > 0$,
$-\pi + \delta_{Vcs} < \theta < -\delta_{Vcs}$

$$\theta = 2\tan^{-1}\left\{\frac{1}{sin(\delta_{Vcs})}\left\{\frac{1 - exp[-(q-b)cos(\delta_{Vcs})]}{1 + exp[-(q-b)cos(\delta_{Vcs})]}\ cos(\delta_{Vcs}) - 1\right\}\right\}$$

$$b = \frac{1}{cos(\delta_{Vcs})}\, ln\left|\frac{-sin(\delta_{Vcs}) + 1 - cos(\delta_{Vcs})}{-sin(\delta_{Vcs}) + 1 + cos(\delta_{Vcs})}\right| \qquad \left(\text{when } q = 0,\ \theta = -\frac{\pi}{2}\right)$$

The figure of case of $\delta_{Vcs} < 0$, it becomes like a shape that Fig.2.3-2 is reversed up and down against the q axis.
Like the chapter2.2, the $\boldsymbol{q}$ in Fig.2.3-2 can be replaced by the distance axis U (=qL) and by the time axis T (=qL/Vcs) when Vcs is constant. Since it is $\varphi = qL/r_{CS}$, it can also be a function of φ.
As using $s_{\overline{cs\text{-}cv}} = -L/sin\theta$ to erase θ, the relationship between $s_{\overline{cs\text{-}cv}}$ and q can be lead.
When CS goes on any curve, by the curvature ρ_{CS}, the target line $\theta = -\delta_{Vcs}$ changes.

Such a motion, when expressed in mathematical formulas, is derived as follows.
When the representative point CS moves on the arc orbit of radius r_{CS} centered on CT,
the coordinate of CS is expressed as follows.

$$\begin{pmatrix} X_{CS} \\ Y_{CS} \end{pmatrix} = r_{CS}\begin{pmatrix} cos\varphi \\ sin\varphi \end{pmatrix} + \begin{pmatrix} X_{CT} \\ Y_{CT} \end{pmatrix}$$

When CT is a constant point and r_{CS} is also a constant, the velocity vector of CS is expressed as follows.

$$\overrightarrow{V_{CS}} = \frac{d}{dt}\begin{pmatrix} X_{CS} \\ Y_{CS} \end{pmatrix} = r_{CS}\frac{d\varphi}{dt}\begin{pmatrix} -sin\varphi \\ cos\varphi \end{pmatrix}$$

If L, θ, $s_{\overline{cs\text{-}cv}}$ are set like the chapter2.1, then the vector from CV to CS is represented as follows.

$$\overrightarrow{CV\text{-}CS} = s_{\overline{cs\text{-}cv}}\begin{pmatrix} cos\varphi \\ sin\varphi \end{pmatrix}$$

The angular velocity of automobile Ω_{CS} is the sum of the rate of change over time of φ and θ.

$$\Omega_{CS} = \frac{d\theta}{dt} + \frac{d\varphi}{dt}$$

When these formulas are applied to the velocity center theorem $\overrightarrow{V_{CS}} = \Omega_{CS}\, Rot\left(\frac{\pi}{2}\right) \overrightarrow{CV\text{-}CS}$, it is as follows.

$$r_{CS}\frac{d\varphi}{dt}\begin{pmatrix}-sin\varphi\\ cos\varphi\end{pmatrix} = \left(\frac{d\theta}{dt}+\frac{d\varphi}{dt}\right)Rot\left(\frac{\pi}{2}\right)\begin{pmatrix}s_{\overline{cs\text{-}cv}}cos\varphi\\ s_{\overline{cs\text{-}cv}}sin\varphi\end{pmatrix}$$

$$= s_{\overline{cs\text{-}cv}}\left(\frac{d\theta}{dt}+\frac{d\varphi}{dt}\right)\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\begin{pmatrix}cos\varphi\\ sin\varphi\end{pmatrix}$$

$$= -\frac{L}{sin\theta}\left(\frac{d\theta}{dt}+\frac{d\varphi}{dt}\right)\begin{pmatrix}-sin\varphi\\ cos\varphi\end{pmatrix}$$

Therefore, the following formula is established.

$$r_{CS}\frac{d\varphi}{dt} = -\frac{L}{sin\theta}\left(\frac{d\theta}{dt}+\frac{d\varphi}{dt}\right)$$

The above equation can be modified as follows.

$$\frac{d\theta}{dt} = -\frac{d\varphi}{dt}\left(1 + r_{CS}\frac{sin\theta}{L}\right) \quad \left(= -\Omega_{V_{CS}} + \Omega_{CS}\right)$$

(When $V_{CS} = r_{CS}\frac{d\varphi}{dt}$、 $\frac{d\varphi}{dt} \to 0$、 $r_{CS} \to +\infty$ 、

the above equation is expressed as the equation of chapter2.2 $\frac{d\theta}{dt} = -V_{CS}\frac{sin\theta}{L}$.)

Therefore, the following formula is established.

$$\frac{d\theta}{d\varphi} = -1 - r_{CS}\frac{sin\theta}{L}$$

Furthermore, like the chapter2.2, U is the distance that CS goes, and the dimensionless quantity q as U/L,
The following relationships will be established.

$$dq = \frac{dU}{L} = \frac{r_{CS}d\varphi}{L} = \frac{d\varphi}{sin\delta_{Vcs}}$$

Therefore,

$$\frac{d\theta}{dq} = \frac{d\theta}{d\varphi}sin\delta_{Vcs} = -\left(1 + r_{CS}\frac{sin\theta}{L}\right)sin\delta_{Vcs} = -sin\delta_{Vcs} - sin\theta$$

When $\delta_{Vcs} = 0$, it is a formula in the case of going straight ahead, as in the previous chapter.、
When $\delta_{Vcs} \neq 0$, the solution of the above formula is as follows.

$$-q + b = \frac{1}{cos(\delta_{Vcs})}ln\left|\frac{sin(\delta_{Vcs})tan\left(\frac{\theta}{2}\right) + 1 - cos(\delta_{Vcs})}{sin(\delta_{Vcs})tan\left(\frac{\theta}{2}\right) + 1 + cos(\delta_{Vcs})}\right| \qquad ※3$$

$$= \frac{1}{cos(\delta_{Vcs})}ln\left|\frac{sin\left(\frac{\theta+\delta_{Vcs}}{2}\right)}{cos\left(\frac{\theta-\delta_{Vcs}}{2}\right)}tan\left(\frac{\delta_{Vcs}}{2}\right)\right|$$

“b” is a constant value that determined by the initial value.

The ± of $\frac{sin\left(\frac{\theta+\delta_{Vcs}}{2}\right)}{cos\left(\frac{\theta-\delta_{Vcs}}{2}\right)}tan\left(\frac{\delta_{Vcs}}{2}\right)$ is equal to the ± of $(sin\theta + sin\delta_{Vcs})sin\delta_{Vcs}$.

When the relationship between θ 、 δ_{Vcs} and $(sin\theta + sin\delta_{Vcs})sin\delta_{Vcs}$ is shown in the table,
it looks like the following Fig.2.3-2.

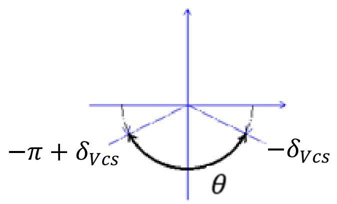
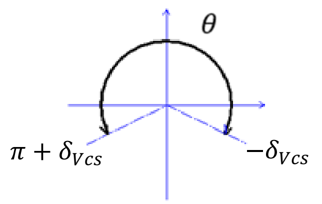
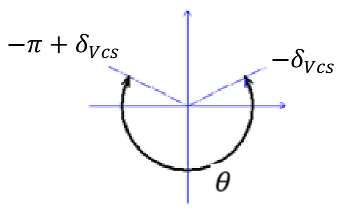
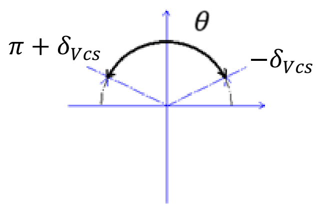
	$-\pi + \delta_{Vcs} < \theta < -\delta_{Vcs}$ $((sin\theta + sin\delta_{Vcs}) < 0)$	$-\delta_{Vcs} < \theta < \pi + \delta_{Vcs}$ $((sin\theta + sin\delta_{Vcs}) > 0)$
$0 < \delta_{Vcs} < \dfrac{\pi}{2}$ $(sin\delta_{Vcs} > 0)$	$(sin\theta + sin\delta_{Vcs})sin\delta_{Vcs} < 0$  	$(sin\theta + sin\delta_{Vcs})sin\delta_{Vcs} > 0$ 
$-\dfrac{\pi}{2} < \delta_{Vcs} < 0$ $(sin\delta_{Vcs} < 0)$	$(sin\theta + sin\delta_{Vcs})sin\delta_{Vcs} > 0$  	$(sin\theta + sin\delta_{Vcs})sin\delta_{Vcs} < 0$ 

Fig. 2.3-2 The relationship between θ 、δ_{Vcs} and $(sin\theta + sin\delta_{Vcs})sin\delta_{Vcs}$

From Fig.2.3-2, the formula ※3 is divided as follows.

When $(sin\theta + sin\delta_{Vcs})sin\delta_{Vcs} > 0$,

$$-q + b = \frac{1}{cos(\delta_{Vcs})} ln\left(\frac{sin(\delta_{Vcs})tan\left(\frac{\theta}{2}\right) + 1 - cos(\delta_{Vcs})}{sin(\delta_{Vcs})tan\left(\frac{\theta}{2}\right) + 1 + cos(\delta_{Vcs})}\right)$$

If θ is expressed as a function of q, then,

$$\theta = 2\tan^{-1}\left\{\frac{1}{sin(\delta_{Vcs})}\left\{\frac{1 + exp[-(q-b)cos(\delta_{Vcs})]}{1 - exp[-(q-b)cos(\delta_{Vcs})]} cos(\delta_{Vcs}) - 1\right\}\right\}$$

When $(sin\theta + sin\delta_{Vcs})sin\delta_{Vcs} < 0$,

$$-q + b = \frac{1}{cos(\delta_{Vcs})} ln\left(-\frac{sin(\delta_{Vcs})tan\left(\frac{\theta}{2}\right) + 1 - cos(\delta_{Vcs})}{sin(\delta_{Vcs})tan\left(\frac{\theta}{2}\right) + 1 + cos(\delta_{Vcs})}\right)$$

If θ is expressed as a function of q, then,

$$\theta = 2\tan^{-1}\left\{\frac{1}{sin(\delta_{Vcs})}\left\{\frac{1 - exp[-(q-b)cos(\delta_{Vcs})]}{1 + exp[-(q-b)cos(\delta_{Vcs})]} cos(\delta_{Vcs}) - 1\right\}\right\}$$

Fig2.3-2 is an example of the following case with the above formula.

$\delta_{Vcs} > 0$ and when q = 0, θ = ± π / 2,

When q = 0 and θ = $+\pi/2$, $b = \frac{1}{cos(\delta_{Vcs})} ln\left|\frac{sin(\delta_{Vcs})+1-cos(\delta_{Vcs})}{sin(\delta_{Vcs})+1+cos(\delta_{Vcs})}\right|$

When q = 0 and θ = $-\pi/2$, $b = \frac{1}{cos(\delta_{Vcs})} ln\left|\frac{-sin(\delta_{Vcs})+1-cos(\delta_{Vcs})}{-sin(\delta_{Vcs})+1+cos(\delta_{Vcs})}\right|$

The specific examples so far are shown next.

The amount of change q required to become θ = 0 to -299π/1800 rad (-29.9°) is represented as next,

$\Delta q_{+} = q_{-299\pi/1800} - q_{0} ≒ 7.9682 - 1.5207 = 6.4475$

Similarly, the amount of change q required to become θ = -π/3 rad (-60°) to -301π/1800 rad (-30.1°) is represented as next,

$\Delta q_{-} = q_{-301\pi/1800} - q_{-\pi/3} ≒ 7.9670 - 1.1605 = 6.8065$

In the case of an orbit with curvature, it can be seen the amount of change q differs depending on whether θ is corrected from inside or outside.

3. Results

By using the velocity center theorem appropriately, it was shown that the automobile's posture can be corrected while the representative point CS goes on an ideal trajectory. By taking the appropriate representative point CS according to the situation, it can be applied to autonomous driving and posture correction when slipping etc.

4. References

[1]「The velocity center theorem of a spinning mobile object and its application to the vehicles」
DOI: https://doi.org/10.51094/jxiv.314