Axisymmetric Magnetic Fields Can Be Stably Generated by Simple Convection

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Researchers have struggled to understand the mechanism underlying the formation of celestial magnetic fields. Ideas in some studies, including Cowling's theory, limit research. Complex convection is necessary to generate a magnetic field. Research in this field could progress through the discovery of a new underlying mechanism. We found a clue in Cowling's paper. In Cowling's theory, to verify the possibility of generating a stable magnetic field, a condition that does not allow the magnetic field to fluctuate is imposed. However, for the magnetic field to increase to a certain intensity, a fluctuating process is essential. Therefore, in this work, we propose a new idea related to this theory. In our theory, the magnetic field and convection are allowed to fluctuate once in the equation. In addition, a necessary term was added to the relational equation based on the concept of external energy replenishment. As a result, even under relatively simple convection, the possibility of generating certain magnetic fields is demonstrated. This is a novel idea, and we believe that these findings will contribute to further elucidation of the mechanism of the formation of celestial magnetic fields.

Keywords: Cowling's theorem, dynamo theory, simple convection, celestial magnetic fields

1. Introduction

Researchers have long struggled to understand the mechanism underlying the formation of celestial magnetic fields. The famous foundations for elucidating the mechanism of the formation of celestial magnetic fields are the ω effect (Levy 1976), the α effect (Parker 1955), and Cowling's theorem (Cowling 1933).

Taking the sun as an example, the magnetic field in the plane perpendicular to the axis of rotation of the sun is called the toroidal magnetic field, and the magnetic field in the plane parallel to the axis of rotation is called the poloidal magnetic field. The same is true for convection.

According to Cowling's theorem, axisymmetric convection does not generate a stable axisymmetric magnetic field either poloidal or toroidal.

The ω effect is the effect of generating a toroidal magnetic field from a poloidal magnetic field where there is a gradient in angular velocity. Since the rotation of the surface of the Sun is faster at the equator than at the poles, there is an angular velocity gradient. If there is a poloidal magnetic field as the initial magnetic field, the magnetic field is stretched so that it is wound up by the angular velocity gradient, and the poloidal magnetic field becomes the toroidal magnetic field. If the toroidal magnetic field is changed to a poloidal

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magnetic field, the magnetic field may be amplified. However, no such effect was found. In the end, the result was in favor of Cowling's theorem.

The α effect assumes a velocity field that twists a magnetic field. The idea is to twist the toroidal magnetic field in some places and direct it in the poloidal direction. Therefore, if an α effect is added to the ω effect, mutual exchange of magnetic fields is possible, and the magnetic field may be amplified. However, this approach is not as easy to use as described above. Researchers are combining these effects with complex convection to further elucidate the mechanism of magnetic field generation. To our knowledge, few papers have argued for the generation of magnetic fields by simple convection.

The notion that a magnetic field is generated by complex convection or that an axisymmetric magnetic field does not occur limits the study. If it is clarified that a magnetic field can be generated by simpler convection, research in this field will further advance. This paper explores the possibility of generating a magnetic field by convection, which is simpler.

Hereafter, unless otherwise stated, the paper by Cowling (1933) is referred to as Cowling's paper. The argument presented in Cowling's paper indicates that it is impossible for an axially symmetric field to be self-maintained (hereinafter also referred to as Cowling's theorem).

This research aims to address this key issue. While we were studying to understand Cowling's paper, we noted a potential issue in Cowling's theory (Cowling 1933). Although the magnetic field must increase during the formation process of the magnetic field, this phenomenon is discussed only in a state where the magnetic field is stable, while cases with unstable magnetic fields are neglected. With further research, we discovered that a mechanism for generating a magnetic field has not been described in Cowling's theory.

This theorem is problematic, as indicated by passage following Cowling's paper (pp. 40–43): The first-order partial derivative of the streamfunction of the magnetic field is zero, but the second-order partial derivative is not zero. These values should match; therefore, applying these results to the magnetic maximum or minimum points (hereinafter referred to as the poles) leads to a contradiction in the electromagnetic induction equation.

This explanation depends on Ohm's law (Reall 2022) (p. 13), which does not include the electromotive force (Reall 2022)(p. 65) due to the vector potential. In Cowling's theory, these conditions are used for the magnetic field to remain stable. However, we believe that there are other ways to pursue a stable magnetic field.

In the method examined in our paper, a term for the electromotive force due to vector potential was added to the electromagnetic induction equation to allow fluctuations in the magnetic field. To facilitate this understanding, we examined the induction term energetically. Then, the concept of external energy replenishment was adopted in the equation. We have found the electromagnetic induction equation by which the generation of the poloidal magnetic field holds. Even if the magnetic field is allowed to rise once, it can be stabilized by a mechanism not described in Cowling's theory as a result. Therefore, an axisymmetric magnetic field may be generated by simple axisymmetric convection. This novel idea conflicts with Cowling's theorem.

2. Description of the Problem

Cowling's theory includes conditions in which the magnetic field must be kept stable. For the magnetic field to reach a certain intensity, it must increase. Therefore, considering only the stable state of the magnetic field is insufficient. Furthermore, if the magnetic field increases due to a factor explained later, the magnetic field may be stably maintained via a mechanism outside those considered in Cowling's theory.

In other words, this process should be examined from a broad perspective that accounts for fluctuations in the magnetic field and convection. Therefore, elements that allow fluctuations (especially increases) in the magnetic field were added to Cowling's theory. Specifically, a term for the electromotive force due to the vector potential was added to the equation. Since it is a typical electromotive force, it is a rather natural formula that considers time. In addition, this equation was examined and adapted to include the replenishment of external energy that causes an increase in the intensity of the magnetic field. We will explain this according to the history of our thinking and speculation, which led us to further improve the equation.

Unless otherwise stated, the symbols or similar symbols with the same meaning as those used in Cowling's paper were used here; these meanings have been transcribed almost verbatim in "". Where there is no explanation, we provide a general interpretation.

"Let ρ denote the density of the gas, and c its (vector) velocity at any point; also let $\boldsymbol{H}$ be the magnetic intensity."

The main problem with Cowling's theorem is as follows. In Cowling's theorem, the equation does not include an electromotive force term (Reall 2022) (p. 65) due to the vector potential $\boldsymbol{A}$ and is calculated by using Ohm's law (Reall 2022) (p. 13) as follows:

$$j = \sigma(c_\wedge \boldsymbol{H} - grad\, V) \tag{2.1}$$

Here, "The electric force on the gas due to its motion in a magnetic field is, in E.M.U., given by $c_\wedge \boldsymbol{H}$; the electrostatic force is $-grad\, V$, where V is the electrostatic potential, which we also suppose measured in E.M.U. Hence, if j is the electric current density and σ is the conductivity of the gas", the operator $\wedge$ is not explained in Cowling's paper. It seems to be a wedge product, but we interpreted it as a vector product.

Cowling's theory describes the above formulation as a steady magnetic field under specific conditions. When the electromotive force term is added, (2.1) can be reformulated as follows:

$$j = \sigma\left(c_\wedge \boldsymbol{H} - \frac{\partial \boldsymbol{A}}{\partial t} - grad\, V\right) \tag{2.2}$$

"Let Oz be taken as the axis of symmetry and let ϖ denote the distance of any point from this axis, so that $\varpi^2 = x^2 + y^2$." We then interpreted x, y and z to mean the value of Cartesian coordinates and their direction. By using the terms derived in Cowling's paper, (2.2) can be reorganized into an electromagnetic induction equation as follows:

$$\frac{\partial \boldsymbol{A}}{\partial t} = \frac{1}{4\pi\varpi\sigma}\left(\frac{\partial^2\psi}{\partial\varpi^2} + \frac{\partial^2\psi}{\partial z^2} - \frac{1}{\varpi}\frac{\partial\psi}{\partial\varpi}\right) - \frac{1}{\rho\varpi^2}\left(\frac{\partial\phi}{\partial\varpi}\frac{\partial\psi}{\partial z} - \frac{\partial\phi}{\partial z}\frac{\partial\psi}{\partial\varpi}\right) \tag{2.3}$$

Here, "There exists a generalisted Stokes stream function ϕ, depending on z and ϖ, such that the components of $\boldsymbol{c}$ parallel to and perpendicular to Oz. ψ is a function of ϖ and z, analogous to Stokes's function; it is such that the total magnetic induction across an area perpendicular to Oz bounded by a circle centered on Oz, which passes through a given point, is equal to the value of $2\pi\psi$ at that point."

If we substitute $\frac{\partial\psi}{\partial z} = 0$, $\frac{\partial\psi}{\partial\varpi} = 0$ and $\frac{\partial^2\psi}{\partial\varpi^2} + \frac{\partial^2\psi}{\partial z^2} \neq 0$ into (2.3),the second term on the right side becomes zero, but the first term is not zero. Therefore, the right side is not zero, and the left side, which is zero and leads to contradiction in Cowling's paper, fluctuates. Thus, even if there is a specific pole with a nonzero second-order partial derivative, (2.3) is satisfied.

The first term on the right side of (2.3) is referred to as the attenuation term, and the second term is referred to as the induction term. This equation can be interpreted as follows: The magnetic flux decreases due to attenuation according to the attenuation term.

According to Cowling's theorem, the left side of (2.3) is zero. This method assumes that the magnetic field does not decrease or increase before the study. If there is an initial magnetic field, even attenuation is not allowed because time is not accounted for in the equation. However, the strength of the magnetic field must decrease or increase during a transitional period until it reaches a certain intensity. Therefore, a formulation that does not allow this growth in a magnetic field is not appropriate. Therefore, in this paper, we introduce a term that permits fluctuations (increases) in the magnetic field on the left side, as shown in (2.3).

However, the induction term is zero, as described above. There is still no element of power generation. We thought it was strange that there was no indication of power generation even though it was an induction term. Therefore, we examine this phenomenon with energetic consideration. The reasons for this are explained below.

We investigated the energetics of this term. Multiplying the parentheses in the second term by ψ yields:

$$\psi\left(\frac{\partial\phi}{\partial\varpi}\frac{\partial\psi}{\partial z}-\frac{\partial\phi}{\partial z}\frac{\partial\psi}{\partial\varpi}\right)=\frac{\partial\phi}{\partial\varpi}\psi\frac{\partial\psi}{\partial z}-\frac{\partial\phi}{\partial z}\psi\frac{\partial\psi}{\partial\varpi} \tag{2.4}$$

Then, when (2.4) is transformed by using partial integrals, the following expression can be obtained:

$$=-\phi\frac{\partial}{\partial\varpi}\left(\psi\frac{\partial\psi}{\partial z}\right)+\phi\frac{\partial}{\partial z}\left(\psi\frac{\partial\psi}{\partial\varpi}\right) \tag{2.5}$$

Furthermore, if we decompose (2.5), the following expression can be obtained:

$$=-\phi\frac{\partial\psi}{\partial\varpi}\frac{\partial\psi}{\partial z}-\phi\psi\frac{\partial^2\psi}{\partial\varpi\partial z}+\phi\frac{\partial\psi}{\partial z}\frac{\partial\psi}{\partial\varpi}+\phi\psi\frac{\partial^2\psi}{\partial z\partial\varpi} \tag{2.6}$$

The first and third terms on the right side of (2.6) are both zero due to the conditions of the poles. The second and fourth terms have the same absolute value but opposite signs; these terms offset one another and become zero. However, each term has its own meaning. Since the polarity of the energy is inverted depending on the condition, it is hereinafter referred to as positive or negative energy corresponding to the effect on the left side of (2.3). At the positive pole, a positive effect on the left side of (2.3) indicates power generation, and a negative effect indicates consumption. To be slightly more specific, power generation (the left side of (2.3) increases) consumes convection energy and electromotion that moves convection consumes electrical energy (the left side of (2.3) decreases). In other words, the same value of energy flows only back and forth at the same time. However, over time, the energy should not come and go forever. Therefore, there is a sense of discomfort associated with this negative energy. Here, we speculated that there is still something missing in (2.3). This will be discussed later. The signs of these two terms may be inverted depending on conditions such as the ϖ and z locations, but the two terms are always offset because they are symmetrically inverted. Therefore, in the above calculation, the induction term is always zero.

Here, we present new ideas about this negative energy. ϕ is a variable similar to ψ, and it fluctuates if time is accounted for in the equation. If this induction term does not account for energy exchange with the environment, both ψ and ϕ decrease over time. However, in reality, convection energy is replenished because if convection continues, the energy must be replenished from the external environment. There is no need for electrical energy to be consumed and converted into convective energy. Therefore, the energy to be replenished should be considered. Thus, the equation should express how power generation occurs without a lack of energy. This negative energy refers to the consumption of energy or the energy used to generate the force that opposes power generation. In the induction term, the replenishment energy is expressed as a force.

The power generation mechanism can be explained by considering the influence of the energy from the environment added to the induction term in (2.3).

$$\frac{\partial \boldsymbol{A}}{\partial t} = \frac{1}{4\pi\varpi\sigma}\left(\frac{\partial^2\psi}{\partial\varpi^2} + \frac{\partial^2\psi}{\partial z^2} - \frac{1}{\varpi}\frac{\partial\psi}{\partial\varpi}\right) - \frac{1}{\rho\varpi^2}\left(\frac{\partial\phi}{\partial\varpi}\frac{\partial\psi}{\partial z} - \frac{\partial\phi}{\partial z}\frac{\partial\psi}{\partial\varpi} + \phi\frac{\partial^2\psi}{\partial\varpi\partial z}\right) \tag{2.7}$$

Specifically, the value of the force obtained by dividing the replenishment energy, $+\phi\psi\frac{\partial^2\psi}{\partial\varpi\partial z}$, equivalent to the negative energy (term 2 in (2.6)) by ψ, $+\phi\frac{\partial^2\psi}{\partial\varpi\partial z}$, in the parentheses of the induction term in (2.3) is used to obtain (2.7). As a result, even at the poles, the induction term does not become zero, and power can be generated while convection receives a supply of energy. Depending on the ϖ and z positions of the poles, the magnetic field strength increases.

Furthermore, when the magnetic field increases, convection energy is converted to electrical energy, and power generation should decrease and stop due to the lack of convection energy. However, since convection energy should always be replenished from the external environment, this process will never stop. However, there should be a limit to the energy supply capacity of convection, so if power generation increases further, energy replenishment will be insufficient, and power generation will be suppressed. In the end, the energy of power generation and supply is balanced somewhere, and the magnetic field stabilizes. Thus, the magnetic field can be maintained at a certain level. The energy supply capacity of convection is not the subject of this paper, and it does not describe where it is balanced.

This result was obtained from a new approach based on the concept of external energy replenishment, with the magnetic field and convection allowed to fluctuate once in our equation. We determined that axisymmetric magnetic fields remain stable at certain intensities.

3. Discussion

For relevant research to progress, a new mechanism underlying the generation of magnetic fields in conductive fluids must be identified. We find that axisymmetric magnetic fields remain stable at certain intensity levels. In addition, convection is required only in the ϖ and z directions. In other words, magnetic field generation occurs by simple poloidal convection. Cowling's paper discusses equations that do not account for changes in magnetic field and convection in time, so it is not possible to think that the magnetic field stabilizes once it rises.

Furthermore, by adding a term for the force corresponding to the replenishment energy, as shown in (2.7), it was found that the energy suitable for power generation can be satisfied, but the flow velocity decreases further as power generation increases. In addition, this additional term also has the side effect of indicating the meaning of the phenomenon. The meaning, which is difficult to see in (2.6), of energy conversion is shown as follows. The value of the second-order partial derivative in this term is determined according to the strength of the magnetic field. This term, multiplied by this value and the streamfunction ϕ, coincides with the characteristics of self-excited power generation, which generates its own magnetic field using the initial magnetic field as a seed and strengthens power generation. Therefore, this term is considered to mean self-excited power generation.

In addition, in the above discussion, neither convection nor magnetic fields require changes due to the direction of rotation of the axis of symmetry. If the convection is uniform in direction and only poloidal, the magnetic field is expected to be uniform in direction and only poloidal.

However, whether the magnetic field is completely stable has not been determined. This is because of the effect of convection deformation due to the force against convection.

If an axisymmetric magnetic field can be generated by simple convection, the conditions for research in related fields may change, which may lead to new research results.

In this study, we examined only the equations for poloidal convection and poloidal magnetic fields, but it is also necessary to consider other cases. In addition, it is desirable to conduct relational research using the results of this theory in various approaches and to verify this theory.

4. Conclusion

Thus, an axisymmetric magnetic field can be generated from simple poloidal convection. We believe that this will provide a great clue to related research.

However, J. J. Love (1996) reported results similar to those of this paper. We speculated that it describes a mechanism that is fundamentally the same as ours. However, the methods and claims are different. The following are some points of concern about the differences from this paper.

First, they searched for the most efficient magnetic Reynolds number R_m for power generation with dipole symmetry. R_m contains a component of flow velocity. In other words, this method obtains a specific flow rate with the highest efficiency. On the other hand, in our theory, the flow velocity fluctuates, and the strength of the magnetic field is determined accordingly.

The other is that there is some difference in convection depending on the azimuth. In addition, the generated magnetic field contains components other than poloidal components, and the value changes depending on the azimuth. On the other hand, according to our theory, convection and magnetic fields are uniform in direction and only poloidal.

It remains to be seen how to interpret these differences. However, additional research is needed.

In any case, we believe that the results of this paper will be of great help in further elucidating the mechanism of the formation of celestial magnetic fields.

Conflicts of Interest
The author has no conflicts of interest to declare.

Acknowledgments

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