Axisymmetric Magnetic Fields can be Stably Generated via Simple Convection

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Abstract

Researchers have struggled to understand the mechanism underlying the formation of celestial magnetic fields. To avoid constraints such as Cowling's theorem, several studies have set up complex convection and developed theories for magnetic field generation. Research in this field could progress through the discovery of a simple mechanism. This paper pursues the power generation theory of a simple mechanism that generates a coaxial magnetic field by axisymmetric convection in a conductive fluid. Whether this power generation can be stably maintained depends on whether there is a source of power generation in the induction term. This problem can be solved by decomposing and examining the induction term in detail. Cowling's theory omits certain components (additional terms) from this term; however, in this study, these components are determined to be essential for axisymmetric magnetic field generation based on calculations and examination of the total amount of regional energy. This approach is a novel concept. In addition, another claim of Cowling's theorem concerning the neutral point is also disputed. Consequently, even under simple axisymmetric convection, the possibility of generating axisymmetric magnetic fields is demonstrated. These findings will contribute to further elucidation of the mechanism of magnetic field formation.

Keywords: Cowling's theorem, dynamo theory, simple convection, neutral point

1 Introduction

Researchers have long struggled to understand the mechanism underlying the formation of celestial magnetic fields. The famous foundations for elucidating this mechanism are the ω effect [1], the α effect [2], and Cowling's theorem [3].

Taking the Sun as an example, the magnetic field in the plane perpendicular to its axis of rotation is called the toroidal magnetic field, and the magnetic field in the plane parallel to its axis of rotation is called the poloidal magnetic field. The same is true for convection.

According to Cowling's theorem, axisymmetric convection does not generate a stable axisymmetric magnetic field, either poloidal or toroidal.

In the ω effect, a toroidal magnetic field is generated from a poloidal magnetic field with a gradient in angular velocity. Since the rotation of the Sun's surface is faster at the equator than at the poles, there is an angular velocity gradient. If the initial magnetic field is poloidal, this field is stretched and wound up by the angular velocity gradient, transforming the poloidal magnetic field into a toroidal magnetic field. If a mechanism exists to convert the toroidal magnetic field to a poloidal magnetic field, then the magnetic field can be amplified. However, no such effect has been found. Thus, the results support Cowling's theorem.

In the α effect, a velocity field that twists a magnetic field is assumed to exist. The concept is that a toroidal magnetic field is twisted in some places and directed in the poloidal direction. Therefore, if the α effect is added to the ω effect, then mutual exchange of magnetic fields is possible, and the magnetic field can be amplified. However, this approach is difficult because it must address complex convection, which causes the velocity field to twist in places. Researchers are nevertheless combining these effects with complex convection to further elucidate the mechanism of magnetic field generation.

The notion that a magnetic field is generated by complex convection or that an axisymmetric magnetic field does not occur constrains research. However, in the observations, the difference between the axis of rotation and the magnetic axis is not large for the main celestial bodies in the solar system, especially for Saturn[4]. If it can be clarified that a magnetic field can be generated by simpler convection, then research in this field will further advance. To reveal a simpler mechanism, this paper[1] addresses the possibility of generating a magnetic field by convection, which is a simpler concept.

We compare our theory with Cowling's theorem and its interpretations, as presented in well-known and authoritative texts. The problem of whether axisymmetric poloidal convection can lead to the formation of stable magnetic fields can be solved by decomposing and examining the induction term ($\boldsymbol{u}_p \times \boldsymbol{B}_p$ described below) in detail. In the original text of Cowling's theorem, components corresponding to the r and z components of convection and magnetic fields are presented, but they do not contain the additional components (hereinafter also referred to as additional terms) that should have emerged when the components are decomposed in cylindrical coordinates. However, in this study, these additional components are essential for axisymmetric magnetic field generation based on calculations and examinations of the total amount of regional energy. This approach is a novel concept. To illustrate these differences, we present the original interpretation of Cowling's theorem and compare it with our new interpretation. The meaning and properties of these additional components (equivalent to self-excited power generation) are described by calculating the total amount of

[1] An earlier version of our original manuscript is available on a preprint server[5].

regional energy. In addition, another claim of Cowling's theorem, the neutral point, is also disputed.

However, this paper does not use the conversion formula in the Curl calculation of vectors, but the objections and others will be discussed in the Discussion section.

2 Description of the Problem

This paper proposes that the mechanism by which axisymmetric magnetic fields are generated from axisymmetric convection should be considered among the fundamental processes for understanding the formation of celestial magnetic fields. We highlight the differences between our findings and conventional theories and explain the key points elucidated in this study. Cowling's theorem, which is the origin of the conventional theorem, has been interpreted by several researchers. For example, [6] and [7] are books that contain these interpretations. These books are cited in the following discussions because the notation of their mathematical formulas is newer and easier to understand.

2.1 Components and Functions of the Induction Term

To identify the components of interest within the electromagnetic induction equation, we decompose the relevant term. We also present the regional integral of energy to investigate the properties of these components. This analysis allows us to describe the potential for generating a simple axisymmetric magnetic field. The electromagnetic induction equation [6] [7] for the poloidal component of a velocity field and a magnetic field, which often serves as the starting point for axisymmetric discussions, is as follows:

$$\frac{\partial \boldsymbol{B}_p}{\partial t} = \nabla \times [\boldsymbol{u}_p \times \boldsymbol{B}_p] + \eta \nabla^2 \boldsymbol{B}_p, \tag{1}$$

where $\boldsymbol{u}_p$ is the flow velocity of the conductive fluid, $\boldsymbol{B}_p$ is the magnetic field, η is the magnetic diffusivity (some researchers use λ) and t is the time. Some researchers use wedge products $\wedge$ for cross-products $\times$, but in this paper, we interpret these symbols as cross-products. The subscript p at the bottom right of a symbol indicates the poloidal component. The magnetic field equation, $\boldsymbol{B}_p = \nabla \times \boldsymbol{A}_\phi$ ($\boldsymbol{A}$ is the vector potential of the magnetic field [8]), is substituted into (1) on the left side as follows:

$$\nabla \times \frac{\partial \boldsymbol{A}_\phi}{\partial t} = \nabla \times [\boldsymbol{u}_p \times \boldsymbol{B}_p] - \nabla \times \eta \nabla \times \boldsymbol{B}_p.$$

Here, the Laplacian of the second term on the right side of (1) is treated in cylindrical coordinates. In addition, Gauss's law for the magnetic field, $\nabla \cdot \boldsymbol{B} = 0$, is applied. The subscript ϕ at the bottom right of a symbol indicates the toroidal component. By uncurling this expression, the following equation is obtained:

$$\frac{\partial \boldsymbol{A}_\phi}{\partial t} = [\boldsymbol{u}_p \times \boldsymbol{B}_p] - \eta \nabla \times \boldsymbol{B}_p. \tag{2}$$

Equations like (2) are derived and are often used as the basis for arguments [6] [7]. Since the left-side and right-side second terms (hereinafter referred to as the attenuation term) of (2) are not the main points here, only the first term on the right side (in this paper, referred to as the induction term) (3) is extracted and examined:

$$\boldsymbol{u}_p \times \boldsymbol{B}_p. \tag{3}$$

By decomposing this term into components in detail, the important components can be identified. The components are decomposed by converting $\boldsymbol{u}_p$ and $\boldsymbol{B}_p$ into descriptions through rotation of the vector potentials. This process is performed in cylindrical coordinates. In other words, each ∇ is treated in cylindrical coordinates. Hereafter, the component directions in the coordinate system are as follows: when the axis of symmetry is called O_z, the direction of distance from O_z is called r, the direction of rotation around O_z is called ϕ, and the direction parallel to O_z is called z. The subscripts at the bottom right of a symbol indicate the directional component. The fundamental vectors in each direction are $\mathbf{e}_r$, $\mathbf{e}_\phi$ and $\mathbf{e}_z$. Furthermore, the electric circuit to which (2) applies circularly orbits O_z in the ϕ direction. Hereafter, it is called a 'ring', and its image corresponds to an annular circuit with a thin cross-sectional area. The component decomposition of $\boldsymbol{u}_p$ is as follows:

$$\boldsymbol{u}_p = \nabla \times P\mathbf{e}_\phi = -\frac{\partial P}{\partial z}\mathbf{e}_r + \frac{1}{r}\frac{\partial}{\partial r}(rP)\,\mathbf{e}_z = -\frac{\partial P}{\partial z}\mathbf{e}_r + \left(\frac{1}{r}P + \frac{\partial P}{\partial r}\right)\mathbf{e}_z. \tag{4}$$

Here, if the divergence of the flow velocity vector $\boldsymbol{u}_p$ is zero (incompressible), then the vector $\boldsymbol{P}\,(= P\mathbf{e}_\phi)$ is the vector potential of the flow velocity. Since we address a poloidal flow, $\boldsymbol{P}$ has only a ϕ component. Notably, the $\frac{1}{r}P$ term is the additional term.

The component decomposition of $\boldsymbol{B}_p$ is as follows:

$$\boldsymbol{B}_p = \nabla \times A\mathbf{e}_\phi = -\frac{\partial A}{\partial z}\mathbf{e}_r + \frac{1}{r}\frac{\partial}{\partial r}(rA)\,\mathbf{e}_z = -\frac{\partial A}{\partial z}\mathbf{e}_r + \left(\frac{1}{r}A + \frac{\partial A}{\partial r}\right)\mathbf{e}_z. \tag{5}$$

Here, the divergence of the magnetic field vector $\boldsymbol{B}_p$ is zero; then, the vector $\boldsymbol{A}\,(= A\mathbf{e}_\phi)$ is the vector potential of the magnetic field $\boldsymbol{B}_p$. Since we address a poloidal magnetic field, $\boldsymbol{A}$ has only a ϕ component. Notably, the $\frac{1}{r}A$ term is the additional term. (4) and (5) are substituted into (3) as follows:

$$\boldsymbol{u}_p \times \boldsymbol{B}_p = \left[-\frac{\partial P}{\partial z}\mathbf{e}_r + \left(\frac{1}{r}P + \frac{\partial P}{\partial r}\right)\mathbf{e}_z\right] \times \left[-\frac{\partial A}{\partial z}\mathbf{e}_r + \left(\frac{1}{r}A + \frac{\partial A}{\partial r}\right)\mathbf{e}_z\right].$$

The following equation is derived:

$$\boldsymbol{u}_p \times \boldsymbol{B}_p = -\left[\left(\frac{1}{r}P + \frac{\partial P}{\partial r}\right)\frac{\partial A}{\partial z} - \frac{\partial P}{\partial z}\left(\frac{1}{r}A + \frac{\partial A}{\partial r}\right)\right]\mathbf{e}_\phi. \tag{6}$$

In the original text of Cowling's theorem, the components corresponding to the r and z components of convection and magnetic fields are presented, but the method of derivation is not shown. Furthermore, the components corresponding to the $\frac{1}{r}P$ and $\frac{1}{r}A$ terms of (6) that should appear when the components are decomposed in cylindrical coordinates are not shown. However, in this study, these additional terms are determined to be essential for axisymmetric magnetic field generation based on calculations and examinations of the total amount of regional energy. These terms can be inferred to possibly be the 'source' of power generation.

The functions and properties of these additional terms are described below. The question is whether this 'source' contributes to power generation. To determine this value, not only one ring but also the rings contained in the entire area must be considered. The total amount of regional energy in these additional terms is calculated and examined. To determine the power generation capacity over the total area, (6) is multiplied by A to convert it into energy.

The $-\frac{1}{r}P\frac{\partial A}{\partial z}$ term in (6) is multiplied by A and partially integrated in the z direction as follows (in the middle of the calculation, $-\frac{1}{r}$ is omitted):

$$\begin{aligned}\int \frac{\partial A}{\partial z}PAdz &= [APA] - \int A\left(\frac{\partial P}{\partial z}A + P\frac{\partial A}{\partial z}\right)dz \\ &= -\int \frac{\partial P}{\partial z}A^2dz - \int PA\frac{\partial A}{\partial z}dz.\end{aligned}$$

Here, P and A in the surface term are zero at $z = \pm\infty$. Since the second term on the right side is the same as that on the left side,

$$\int \frac{\partial A}{\partial z}PAdz = -\frac{1}{2}\int \frac{\partial P}{\partial z}A^2dz. \tag{7}$$

Since the integration over ϕ is uniform at the circumference, this equation is multiplied by $2\pi r$. Adding $-\frac{1}{r}$ back in and integrating it over r yields

$$\iiint -\frac{1}{r}PA\frac{\partial A}{\partial z}dV = \frac{1}{2}2\pi r\iint \frac{1}{r}\frac{\partial P}{\partial z}A^2drdz = \pi r\iint \frac{1}{r}\frac{\partial P}{\partial z}A^2drdz, \tag{8}$$

where V denotes the total volume.

The $\frac{\partial P}{\partial z}\frac{1}{r}A$ term in (6) is multiplied by A and integrated over the total area as follows:

$$\iiint \frac{\partial P}{\partial z}\frac{1}{r}AAdV = 2\pi r\iint \frac{1}{r}\frac{\partial P}{\partial z}A^2drdz, \tag{9}$$

Here, since the integration over ϕ is uniform at the circumference, this equation is multiplied by $2\pi r$.

2.2 Significance of the Regional Integration of Energy

Fig. 1 is a conceptual diagram showing how the initial magnetic field moves due to convection. The convection flows in the Z and Ra directions (parallel to the Z axis and radial direction from the Z axis, respectively) in an axisymmetric manner, with the Z axis as the axis of symmetry in the range indicated by the convection zone. The convection is roughly torus shaped with a flow velocity $|\boldsymbol{U}|$ to orbit around P_0 in the direction of the arrow. P_0 is a position in which a line is drawn perpendicular to the Z axis from Pc. The long arrow in (a) and the lattice-like dots in (b) and (c) in Fig. 1 indicate the initial magnetic field. In conductive fluids, the magnetic field moves with the flow of the fluid due to magnetic field freezing[9]. (a) Diagram of convection and

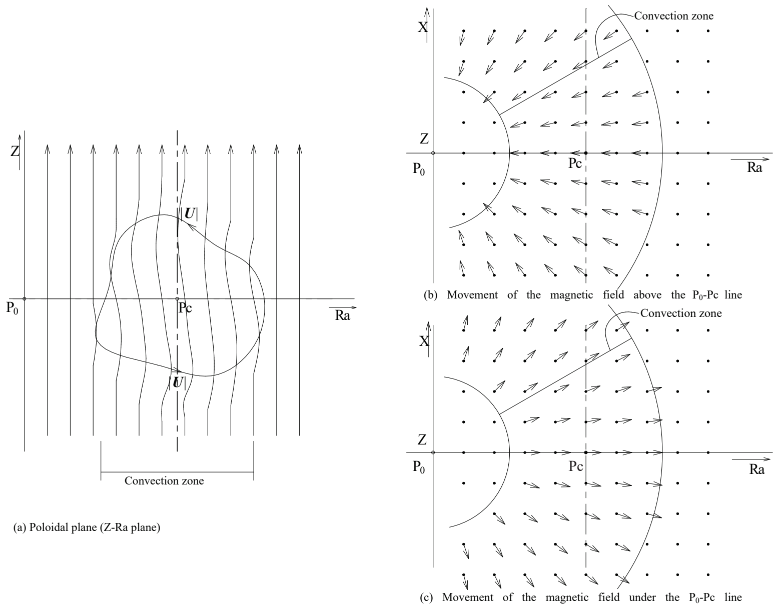

Fig. 1 Conceptual diagram showing how a magnetic field moves due to convection
The convection flows in the Z and Ra directions in an axisymmetric manner, with the Z axis as the axis of symmetry in the range indicated by the convection zone. Convection is roughly torus shaped with a flow velocity $|U|$ to orbit around Pc in the direction of the arrow. P_0 is a position where Pc intersects perpendicular to the Z axis. (a) Diagram in which the existing magnetic field bends on the poloidal surface of Z-Ra, (b) diagram in which the magnetic field moves above the P_0-Pc line, and (c) diagram in which the magnetic field moves below the P_0-Pc line. Since convection is an axisymmetric poloidal flow, this convection converges towards the Z axis in (b) and disperses from the Z axis in (c). The magnetic field also converges and discrete, due to magnetic field freezing. That is, the magnetic flux density increases in (b) and decreases in (c).

the magnetic field of the poloidal surface of Z-Ra, (b) diagram in which convection and the magnetic field move together like a short arrow above the P_0-Pc line, and (c) diagram in which convection and the magnetic field move together like a short arrow below the P_0-Pc line. Since convection is an axisymmetric poloidal flow, this convection converges towards the Z axis in (b) and disperses from the Z axis in (c). Since the magnetic field also converges and disperses due to magnetic field freezing, the magnetic flux density increases in (b) and decreases in (c).

If the energy integral of equations (8) and (9) has a positive polarity value, the energy of the magnetic field is considered to increase. In other words, the energy increases because a magnetic field in the same direction as the existing magnetic field direction is added, which means self-excited power generation. Considering these equations, $\frac{\partial P}{\partial z}$ and A^2 are multiplied and integrated. The former is derived from the velocity of convection in the Ra direction, $u_r = -\frac{\partial P}{\partial z}$ (see equation (4)). In Fig. 1 (b), u_r is the negative direction of Ra ($\frac{\partial P}{\partial z}$ is positive polarity) above the P_0-Pc line, and in (c), it is the positive direction of Ra under the P_0-Pc line ($\frac{\partial P}{\partial z}$ is negative polarity). The flow velocity is symmetrical and has a reverse polarity above and below the P_0-Pc line. If A^2 is symmetrical above and below the P_0-Pc line, then integrating the multiplication with the flow velocity becomes zero, and there is no increase in energy. However, (b) in Fig. 1 has a higher magnetic flux density (vector potential A as well) than (c). Therefore, the positive polarity of $\frac{\partial P}{\partial z}A^2$ of (b) becomes dominant, and the positive energy is calculated. If the flow velocity u_p is sufficient, this energy causes equation (6) $(u_p \times B_p)$ to overcome the attenuation term of equation (2) and self-excited power generation occurs.

Therefore, the region integral of (8) and (9) is nonzero and can be the 'source' of power generation. However, this 'source' does not indicate the stability of the magnetic field but rather an increase in power generation. If the current intensifies, the Lorentz force generates a force that opposes convection. It is thought that the increase in power generation is suppressed and settles at a certain level. However, the behaviour of convection is not accurately considered in (4). Convection behaviour must be considered to confirm these maintenance mechanisms of the magnetic field, but this issue is beyond the scope of this paper.

2.3 Causes of Power Generation

In equations (4) and (5), the terms $\frac{1}{r}P$ and $\frac{1}{r}A$ exist. That is, when handled in cylindrical coordinates, the P and A terms are inversely proportional to the diameter from the Z axis. Referring to the arrangement of (b) and (c) in Fig. 1, which is a cylindrical coordinate, these terms seem to represent the change in density that occurs when a fluid of a certain flow rate and a magnetic field of a certain number of magnetic fluxes move poloidally between a narrow region close to the Z axis and a wide area far from the Z axis. These density changes are thought to be important for electrical phenomena. Therefore, it is necessary to confirm whether these terms contribute to power generation. The energy integrals were shown earlier is because we assume that these terms are important causes of power generation. If these terms are ignored, there is no prospect of generating electricity. When performing axisymmetric and poloidal verification, it is essential to treat it strictly in cylindrical coordinates and to not omit these terms. The original text of Cowling's theorem does not mention these terms. Therefore, Cowling's theorem led to the conclusion that an axisymmetric magnetic field is not maintained from axisymmetric convection.

Therefore, additional terms are found when the induction term is decomposed in detail. When examined in terms of the regional integration of energy, these terms are demonstrated to be the 'source' of power generation. In other words, these terms simply generate a stable axisymmetric magnetic field.

2.4 On the Neutral-point Claim

The original text of Cowling's theorem and its interpretative studies not only deny the source of power generation but also examine the local poloidal magnetic field arrangement (N-point) and deny the existence of axisymmetric magnetic fields. If this paper affirms the source of power generation, it must also be able to affirm the existence of an axisymmetric magnetic field in the N-point claim. Let us start with the conventional theory.

If a poloidal magnetic field exists on a z-r surface, then a vortex centre of the magnetic field must be somewhere, known as the neutral point, or the N-point. This description is explained in textbooks [6] [7]. Note that in the original text of Cowling's theorem, the name of this claim, the symbols used, and the method of explanation are different. (2) serves as the basis for calculations. Since the magnetic field arrangement of the stable magnetic field is discussed, the left side of (2) is zero, and since a certain area on the poloidal surface is addressed, both terms are multiplied by the area $\boldsymbol{S}_\varepsilon$ on the z-r surface and deformed as follows:

$$\int_{S_\varepsilon} (\boldsymbol{u}_p \times \boldsymbol{B}_p) \cdot d\boldsymbol{S} = \int_{S_\varepsilon} \eta \, (\nabla \times \boldsymbol{B}_p) \cdot d\boldsymbol{S}. \tag{10}$$

If we apply Stokes' theorem [8] to the right side of (10), then

$$\int_{S_\varepsilon} (\boldsymbol{u}_p \times \boldsymbol{B}_p) \cdot d\boldsymbol{S} = \oint_{C_\varepsilon} \eta \boldsymbol{B}_p \cdot d\boldsymbol{x}. \tag{11}$$

Here, S_ε is the area inside the line integral on the right side of (11), and C_ε is the circle of the line integral. The approximate values on the left side of (11) are as follows:

$$\left| \int_{S_\varepsilon} (\boldsymbol{u}_p \times \boldsymbol{B}_p) \cdot d\boldsymbol{S} \right| \leq U B_\varepsilon S_\varepsilon. \tag{12}$$

Here, the average magnetic field in the line integration on the right side of (11) is B_ε. The area of each integral is the same. U is the maximum flow velocity in the area. Since the magnetic field is zero[2] in the centre of the N-point, the mean magnetic field within S_ε is smaller than B_ε, but at (12), it is estimated to be B_ε. Then, the relationship between the magnitude of the values on the right side and the left side of (11) is $\eta_N \varepsilon B_\varepsilon \leq U B_\varepsilon S_\varepsilon$ or

$$\eta_N \varepsilon \leq U S_\varepsilon, \tag{13}$$

where ε on the left side is the perimeter of the N-point area and where η_N is the η around the N-point. (To contrast (13) with (11), it may be easier to understand if the left and right sides are reversed, i.e., $US_\varepsilon \geq \eta_N \varepsilon$.) However, since $\varepsilon \to 0$ results in $S_\varepsilon \to \mathrm{O}(\varepsilon^2)$, (13) is not compatible for any finite values $\frac{U}{\eta_N}$. Therefore, it is conventionally thought that this magnetic field arrangement is contradictory, so an axisymmetric magnetic field does not occur.

However, for the following fundamental reasons, this paper refutes the N-point claim, which is unrelated to the claim in Section 2.1. First, the right side of (11) is the right side of (10) according to Stokes' theorem. That is, (11) is an area integral on the left and right sides. (11) shows that the left and right sides are always the same. Applying different order changes to the areas on the left and right sides is incorrect and misleading. Therefore, the N-point claim is invalid.

[2]The centre of the vortex of the magnetic field should have an electric current and the magnetic field becomes stronger towards the centre, it is convex rather than concave. However, the conclusion is the same in both cases.

3 Discussion

We have shown that an axisymmetric magnetic field can be generated by axisymmetric convection and that magnetic field generation occurs through simple poloidal convection. However, this demonstration does not fully explain the stable generation of a magnetic field. In addition, this paper does not use the conversion formula in the Curl calculation of vectors. These thoughts are described below.

The 'source' of power generation does not indicate the stability of the magnetic field but rather an increase in power generation. However, if the current intensifies, the Lorentz force generates a force that opposes convection. It is thought that the increase in power generation is suppressed and settles at a certain level. However, the behaviour of convection is not accurately considered in (4). Convection behaviour must be considered to confirm these maintenance mechanisms of the magnetic field, but this issue is beyond the scope of this paper.

Previously, one peer review included the proposal to convert (5) with the following formula (hereinafter referred to as the conversion formula):

$$\nabla \times (A_\phi e_\phi) = \nabla \times (rA_\phi \nabla\phi) \tag{14}$$

where e_ϕ is the unit vector along ϕ, r is the cylindrical radius and $\nabla\phi$ is the gradient of the angle ϕ

$$e_\phi = r\nabla\phi \,, \psi = rA_\phi \tag{15}$$

$$\nabla \times (A_\phi e_\phi) = (\nabla\psi) \times (\nabla\phi) \tag{16}$$

Moreover, there was an example of a website[10] that was converted by a similar formula. For the following reasons, the conversion formula is not used in this paper. Converting the left side of equation (16) to the right side is clearly different from the treatment of Curl in cylindrical coordinates in vector mathematics. If you treat it in Cartesian coordinates, this conversion formula can be derived. The theory of axisymmetry must be discussed strictly in cylindrical coordinates. If the components are decomposed directly with Curl in cylindrical coordinates, it is easy, and there are no mistakes. The original text of Cowling's theorem does not show this conversion. Moreover, we are unable to find a source explaining the necessity and validity of this conversion. As shown in Section 2.3, Causes of Power Generation, additional terms are shown to be important. If this conversion formula is used, the change in density due to convection and magnetic fields moving back and forth between a narrow and large area is ignored, and an important electrical phenomenon is missed.

4 Conclusion

This paper presents a theory in which axisymmetric magnetic fields are stably generated by simple convection. In addition, another claim of Cowling's theorem concerning the neutral point is also disputed.

Declarations

Conflicts of interest

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Data availability statement

We do not analyse or generate any datasets because our work proceeds via a theoretical and mathematical approach.

Clinical trial

Not applicable

Ethics, consent to participate, and consent to publish declarations

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Author contribution statement

Mamoru Otsuki independently conducted this study and submission. He wrote the main manuscript text and prepared all the figures. Acknowledgements

The study was conducted by only one individual, and there were no direct collaborators. However, we used services such as pre-peer review by an editing company, Microsoft's document processing software and its translation function and draughting software and Adobe Acrobat to draw the diagram. We are grateful for these services and products, which encouraged us to perform this research. We are also grateful for today's IT world, which offers advanced services.

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