Axisymmetric Magnetic Fields Can Be Stably Generated by Simple Convection

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Abstract

Researchers have struggled to understand the mechanism underlying the formation of celestial magnetic fields. In some studies, the conception of this mechanism, such as Cowling's theorem, which posits that complex convection is necessary to generate a magnetic field, constrains research. Research in this field could progress through the discovery of a simple mechanism. To reveal a possible simple mechanism, this paper addresses generating a magnetic field by convection, which is a simple concept. Whether an axisymmetric poloidal magnetic field can be stably generated by the axisymmetric poloidal convection of a conductive fluid depends on whether the induction term contains a source of power generation. This problem can be solved by decomposing and examining the induction term in detail. Cowling's theory omits certain components (terms) from this term; however, in this study, these components are found to be essential for axisymmetric magnetic field generation on the basis of calculations and examination of the total amount of energy. This approach is a novel concept. In addition, another claim of Cowling's theorem concerning the neutral point is also disputed. Consequently, even under simple axisymmetric convection, the possibility of generating axisymmetric magnetic fields is demonstrated. These findings will contribute to further elucidation of the mechanism of magnetic field formation. However, in this work, unresolved points are found in the maintenance of the magnetic field and the deformation of the formula.

Keywords: Cowling's theorem, dynamo theory, simple convection, neutral point

1 Introduction

Researchers have long struggled to understand the mechanism underlying the formation of celestial magnetic fields. The famous foundations for elucidating this mechanism are the ω effect [1], the α effect [2], and Cowling's theorem [3].

Taking the Sun as an example, the magnetic field in the plane perpendicular to its axis of rotation is called the toroidal magnetic field, and the magnetic field in the plane parallel to its axis of rotation is called the poloidal magnetic field. The same is true for convection.

According to Cowling's theorem, axisymmetric convection does not generate a stable axisymmetric magnetic field, either poloidal or toroidal.

In the ω effect, a toroidal magnetic field is generated from a poloidal magnetic field with a gradient in angular velocity. Since the rotation of the Sun's surface is faster at the equator than at the poles, there is an angular velocity gradient. If the initial magnetic field is poloidal, this field is stretched and wound up by the angular velocity gradient, transforming the poloidal magnetic field into a toroidal magnetic field. If a mechanism exists to convert the toroidal magnetic field to a poloidal magnetic field, then the magnetic field can be amplified. However, no such effect has been found. Thus, the results support Cowling's theorem.

In the α effect, a velocity field that twists a magnetic field is assumed to exist. The concept is that a toroidal magnetic field is twisted in some places and directed in the poloidal direction. Therefore, if the α effect is added to the ω effect, then mutual exchange of magnetic fields is possible, and the magnetic field can be amplified. However, this approach is difficult because it must address complex convection that causes the velocity field to twist in places. Researchers are nevertheless combining these effects with complex convection to further elucidate the mechanism of magnetic field generation.

From the above discussion, researchers must rely only on complex convection without a simple magnetic field generation mechanism. The notion that a magnetic field is generated by complex convection or that an axisymmetric magnetic field does not occur constrains research. However, in the observations, the difference between the axis of rotation and the magnetic axis is not large for the main celestial bodies in the solar system, especially for Saturn[4]. If it can be clarified that a magnetic field can be generated by simpler convection, then research in this field will further advance. To reveal a simpler mechanism, this paper addresses the possibility of generating a magnetic field by convection, which is a simpler concept.

We compare our theory with Cowling's theorem and its interpretations, as presented in well-known and authoritative texts. The problem of whether axisymmetric poloidal convection can lead to the formation of stable magnetic fields can be solved by decomposing and examining the induction term in detail. In the original text of Cowling's theorem, components corresponding to the r and z components of convection and magnetic fields are presented, but they do not contain the minute components that should have emerged when the components are decomposed in cylindrical coordinates. However, in this study, these minute components are essential for axisymmetric magnetic field generation on the basis of calculations and examinations of the total amount of energy. This approach is a novel concept. To illustrate these differences, we present the original interpretation of Cowling's theorem and compare it with our new interpretation. The meaning and properties of these minute components (equivalent to self-excited power generation) are described by calculating the total amount of regional energy. In addition, another claim of Cowling's theorem, the neutral point, is also disputed.

However, our theory does not fully explain the stable generation of a magnetic field. In addition, the deformation of the formula contains unresolved points. The limitations of our theory are addressed in the Discussion section. Although definitive evidence for these findings is lacking, we believe that our theory is valuable and outline our expectations in the Conclusion section.

Conventionally, there is no simple generation of a stable axisymmetric poloidal magnetic field from axisymmetric poloidal convection. The theory presented in this paper is a novel concept that challenges conventional thinking.

2 Description of the Problem

This paper proposes that the mechanism by which axisymmetric magnetic fields are generated from axisymmetric convection should be considered among the fundamental processes for understanding the formation of celestial magnetic fields. We highlight the differences between our findings and conventional theories and explain the key points elucidated in this study. Cowling's theorem, which is the origin of the conventional theorem, has been interpreted by several researchers. For example, [5] and [6] are books that contain these interpretations. These books are cited in the following discussions because the notation of their mathematical formulas is newer and easier to understand.

2.1 Components and Functions of the Induction Term

To identify the components of interest within the electromagnetic induction equation, we decompose the relevant term. We also present the regional integral of energy to investigate the properties of these components. This analysis allows us to describe the potential for generating a simple axisymmetric magnetic field. The electromagnetic induction equation [5] [6] for the poloidal component of a velocity field and a magnetic field, which often serves as the starting point for axisymmetric discussions, is as follows:

$$\frac{\partial \boldsymbol{B}_p}{\partial t} = \nabla \times [\boldsymbol{u}_p \times \boldsymbol{B}_p] + \eta \nabla^2 \boldsymbol{B}_p, \tag{1}$$

where $\boldsymbol{u}_p$ is the flow velocity of the conductive fluid, $\boldsymbol{B}_p$ is the magnetic field, η is the magnetic diffusivity (some researchers use λ) and t is the time. Some researchers use wedge products $\wedge$ for cross-products $\times$, but in this paper, we interpret them as cross-products. The subscript p at the bottom right of a symbol indicates the poloidal component. The magnetic field equation, $\boldsymbol{B}_p = \nabla \times \boldsymbol{A}_\phi$ ($\boldsymbol{A}$ is the vector potential of the magnetic field [7]), is substituted into (1) on the left side as follows:

$$\nabla \times \frac{\partial \boldsymbol{A}_\phi}{\partial t} = \nabla \times [\boldsymbol{u}_p \times \boldsymbol{B}_p] - \nabla \times \eta \nabla \times \boldsymbol{B}_p.$$

Here, the Laplacian of the second term on the right side of (1) is treated in cylindrical coordinates. In addition, Gauss's law for the magnetic field, $\nabla \cdot \boldsymbol{B} = 0$, is applied. The subscript ϕ at the bottom right of a symbol indicates the toroidal component. By uncurling this expression, the following equation is obtained:

$$\frac{\partial \boldsymbol{A}_\phi}{\partial t} = [\boldsymbol{u}_p \times \boldsymbol{B}_p] - \eta \nabla \times \boldsymbol{B}_p. \tag{2}$$

Equations similar to (2) are derived and are often used as the basis for arguments [5] [6]. Since the left-side and right-side second terms (hereinafter referred to as the attenuation term) of (2) are not the main points here, only the first term on the right side (in this paper, referred to as the induction term) (3) is extracted and examined:

$$\boldsymbol{u}_p \times \boldsymbol{B}_p. \tag{3}$$

By decomposing this term into components in detail, the important components can be identified. The components are decomposed by converting $\boldsymbol{u}_p$ and $\boldsymbol{B}_p$ into descriptions through rotation of the vector potentials. This process is performed in cylindrical coordinates. In other words, each ∇ is treated in cylindrical coordinates. Hereafter, the component directions in the coordinate system are as follows: when the axis of symmetry is called O_z, the direction of distance from O_z is called r, the direction of rotation around O_z is called ϕ, and the direction parallel to O_z is called z. The subscripts at the bottom right of a symbol indicate the directional component. The fundamental vectors in each direction are $\mathbf{e}_r$, $\mathbf{e}_\phi$ and $\mathbf{e}_z$. Furthermore, the electric circuit to which (2) applies circularly orbits O_z in the ϕ direction. Hereafter, it is called a 'ring', and its image corresponds to an annular circuit with a thin cross-sectional area. The component decomposition of $\boldsymbol{u}_p$ is as follows:

$$\boldsymbol{u}_p = \nabla \times P\mathbf{e}_\phi = -\frac{\partial P}{\partial z}\mathbf{e}_r + \frac{1}{r}\frac{\partial}{\partial r}(rP)\,\mathbf{e}_z = -\frac{\partial P}{\partial z}\mathbf{e}_r + \left(\frac{1}{r}P + \frac{\partial P}{\partial r}\right)\mathbf{e}_z. \tag{4}$$

Here, if the divergence of the flow velocity vector $\boldsymbol{u}_p$ is zero (incompressible), then the vector $\boldsymbol{P}\,(= P\mathbf{e}_\phi)$[1] is the vector potential of the flow velocity. Since we are addressing a poloidal flow, $\boldsymbol{P}$ only has a ϕ component. Notably, the $\frac{1}{r}P$ term is the minute component. When the components are decomposed by Cartesian coordinates, the $\frac{1}{r}P$ term in (4) does not appear.

The component decomposition of $\boldsymbol{B}_p$ is as follows:

$$\boldsymbol{B}_p = \nabla \times A\mathbf{e}_\phi = -\frac{\partial A}{\partial z}\mathbf{e}_r + \frac{1}{r}\frac{\partial}{\partial r}(rA)\,\mathbf{e}_z = -\frac{\partial A}{\partial z}\mathbf{e}_r + \left(\frac{1}{r}A + \frac{\partial A}{\partial r}\right)\mathbf{e}_z. \tag{5}$$

Here, the divergence of the magnetic field vector $\boldsymbol{B}_p$ is zero; then, the vector $\boldsymbol{A}\,(= A\mathbf{e}_\phi)$ is the vector potential of the magnetic field $\boldsymbol{B}_p$. Since we are addressing a poloidal magnetic field, $\boldsymbol{A}$ only has a ϕ component. Notably, the $\frac{1}{r}A$ term is the minute component. When the components are decomposed by Cartesian coordinates, the $\frac{1}{r}A$ term in (5) does not appear. (4) and (5) are substituted into (3) as follows:

[1]Note that distinguishing the difference between P and $\boldsymbol{P}$ in printing is difficult: P in the equation is a scalar and $P\mathbf{e}\phi$ is the ϕ component vector. The same applies to A and $\boldsymbol{A}$ below.

$$\boldsymbol{u}_p \times \boldsymbol{B}_p = \left[-\frac{\partial P}{\partial z}\mathbf{e}_r + \left(\frac{1}{r}P + \frac{\partial P}{\partial r} \right)\mathbf{e}_z \right] \times \left[-\frac{\partial A}{\partial z}\mathbf{e}_r + \left(\frac{1}{r}A + \frac{\partial A}{\partial r} \right)\mathbf{e}_z \right].$$

The following equation is derived:

$$\boldsymbol{u}_p \times \boldsymbol{B}_p = -\left[\left(\frac{1}{r}P + \frac{\partial P}{\partial r} \right)\frac{\partial A}{\partial z} - \frac{\partial P}{\partial z}\left(\frac{1}{r}A + \frac{\partial A}{\partial r} \right) \right]\mathbf{e}_\phi. \tag{6}$$

If (6) is substituted into (2), the direction component of the induction term can be seen in detail. In the original text of Cowling's theorem, the components corresponding to the r and z components of convection and magnetic fields are presented, but the method of derivation is not shown. Furthermore, the components corresponding to the $\frac{1}{r}P$ and $\frac{1}{r}A$ terms of (6) that should appear when the components are decomposed in cylindrical coordinates are not shown. However, in this study, these terms are found to be essential for axisymmetric magnetic field generation on the basis of calculations and examinations of the total amount of energy. These terms are referred to as additional terms and can be inferred to possibly be the 'source' of power generation.

The functions and properties of these additional terms are described below. The question is whether this 'source' contributes to power generation. To determine this value, not only one ring but also the rings contained in the entire area must be considered. The total amount of energy in these additional terms is calculated and examined. To determine the power generation capacity over the total area, (6) is multiplied by A to convert it into energy.

The $-\frac{1}{r}P\frac{\partial A}{\partial z}$ term in (6) is multiplied by A and partially integrated in the z direction as follows (in the middle of the calculation, $-\frac{1}{r}$ is omitted):

$$\begin{aligned} \int \frac{\partial A}{\partial z} PAdz &= [APA] - \int A\left(\frac{\partial P}{\partial z}A + P\frac{\partial A}{\partial z} \right)dz \\ &= -\int \frac{\partial P}{\partial z}A^2 dz - \int PA\frac{\partial A}{\partial z}dz. \end{aligned}$$

Here, P and A in the surface term are zero at $z = \pm\infty$. Since the second term on the right side is the same as that on the left side,

$$\int \frac{\partial A}{\partial z} PAdz = -\frac{1}{2}\int \frac{\partial P}{\partial z}A^2 dz. \tag{7}$$

Since the integration over ϕ is uniform at the circumference, this equation is multiplied by $2\pi r$. Adding $-\frac{1}{r}$ back in and integrating it over r yields

$$\iiint -\frac{1}{r}PA\frac{\partial A}{\partial z}dV = \frac{1}{2}2\pi r \iint \frac{1}{r}\frac{\partial P}{\partial z}A^2 drdz = \pi r \iint \frac{1}{r}\frac{\partial P}{\partial z}A^2 drdz, \tag{8}$$

where V denotes the total volume. Consider what happens to this energy.

Suppose that a torus-like fluid, as shown in Fig. 1, rotates in the $\boldsymbol{U}$ direction. Then, $-\frac{\partial P}{\partial z}$ (the flow velocity u_r in the r direction) is nonzero above and below the r-axis (the line connecting P_0 and P_c), the velocity is symmetric about this axis, and the polarity is opposite on the two sides of this axis. If A^2 is also symmetric about

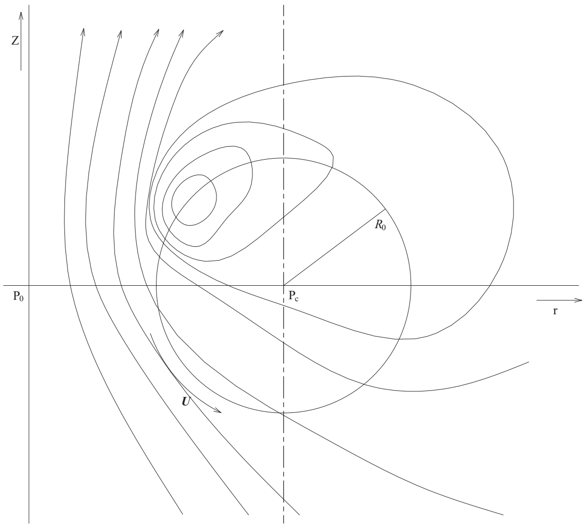

Fig. 1 Conceptual diagram of the flow velocity and magnetic field arrangement on the z-r surface This figure shows that the value A of the vector potential is not linearly symmetric about the r-axis, and the neutral point is outlined. Here, the r-axis is a line in the r direction connecting P_0 and P_c. Convection is the flow of a torus-shaped (doughnut-like) fluid, and this figure shows one side of the cross-section containing the Z-axis, which is the axis of the torus. The convection is drawn in a circle around P_c. A torus-shaped fluid with a convection radius (distance from P_c to the convection surface) of size R_0 rotates in the flow velocity $\boldsymbol{U}$ direction. The surrounding curves represent the path of a magnetic field. The neutral point is considered the location where the main current (power generation) is generated and is convex.

this axis, it will cancel out and become zero. However, if the magnetic field (also in A) is asymmetric about this axis, the region's integration of (8) will not be zero. Thus, energy is generated. The magnetic field in Fig. 1 is an imaginary diagram based on the current (eigenvector) data from the preprint [8] provided as a numerical calculation example of the fluid magnetic field.

The $\frac{\partial P}{\partial z}\frac{1}{r}A$ term in (6) is multiplied by A and integrated over the total area as follows:

$$\iiint \frac{\partial P}{\partial z}\frac{1}{r}AAdV = 2\pi r\iint \frac{1}{r}\frac{\partial P}{\partial z}A^2 drdz, \tag{9}$$

Here, since the integration over ϕ is uniform at the circumference, this equation is multiplied by $2\pi r$.

In (9), since A^2 is asymmetric about the r-axis, as described above, energy is also generated during the integration of this term.

Therefore, the region integral of (8) and (9) is nonzero and can be the 'source' of power generation. However, this 'source' does not indicate the stability of the magnetic field but rather an increase in power generation. However, if power generation is greatly increased, convection energy should be consumed, and the flow velocity should decrease. Therefore, it may be stabilised at a certain magnetic field strength. Another factor that stabilises the magnetic field (power generation) is considered. If the current intensifies, the Lorentz force generates a force that opposes convection. It is thought that the increase in power generation is suppressed and settles at a certain level. However, the behaviour of convection is not accurately considered in (4). Convection behaviour must be considered to confirm these maintenance mechanisms of the magnetic field, but this issue is beyond the scope of this paper.

However, there is one more problem: if the current (power generation) position continues to move along the convection path due to magnetic field freezing [9], power generation may not be possible because of insufficient power generation time in regions where power generation is active. Magnetic field freezing occurs when a magnetic field constrains a fluid with high conductivity. In this case, the above explanation of the 'source' of power generation cannot be given.

However, the magnetic field arrangement in Fig. 1 is considered stable without movement. This thought is held because electromagnetic induction [7] occurs between the rings that move along the convection path. Its function is explained on the basis of Lenz's law[7] as follows: suppose that power generation occurs in a ring at a certain position and that there is a current vertex. Owing to convection, the ring moves along an upwards slope towards its vertex and a downwards slope away from the vertex. On the upwards slope, the current increases, causing the magnetic field to increase. According to Lenz's law, an electric current is generated towards a decreasing magnetic field in parts of the rings. This phenomenon is called electromagnetic induction. Additionally, on the downwards slope, the current decreases, leading to a reduction in the magnetic field. According to Lenz's law, an electric current is generated towards the increasing magnetic field in parts of the rings. Consequently, electromagnetic induction mutually occurs. Then, so that the effects on both sides are commensurate to some extent, the current in the ring moving along the downwards slope acts as if it were transferred to the ring moving along the upwards slope, causing a certain amount of current to remain at a specific position. In this way, neither the power generation position nor the magnetic field moves from a specific position.

The conventional concept that there is no 'source' of magnetic field generation in the induction term is shown in Appendix A (Conventionally, No Source of Power Generation).

Therefore, additional terms are found when the induction term is decomposed in detail. When examined in terms of the regional integration of energy, it is demonstrated that they are the 'source' of power generation. In other words, these terms simply generate a stable axisymmetric magnetic field.

2.2 On the Neutral-point Claim

The original text of Cowling's theorem and its interpretative studies not only deny the source of power generation but also examine the local poloidal magnetic field arrangement (N-point) and deny the existence of axisymmetric magnetic fields. If this paper affirms the source of power generation, it must also be able to affirm the existence of an axisymmetric magnetic field in the N-point claim. Let us start with the conventional theory.

If a poloidal magnetic field exists on a z-r surface, then a vortex centre of the magnetic field must be somewhere, known as the neutral point, or the N-point. The vortex can be imagined as an inverted cone with a magnetic field moving towards zero as it approaches the centre. This description is explained in textbooks [5] [6]. Note that in the original text of Cowling's theorem, the name of this claim, the symbols used, and the method of explanation are different. (2) serves as the basis for calculations. Since the magnetic field arrangement of the stable magnetic field is discussed, the left side of (2) is zero, and since a certain area on the poloidal surface is addressed, both terms are multiplied by the area $\boldsymbol{S}_\varepsilon$ on the z-r surface and deformed as follows:

$$\int_{S_\varepsilon} (\boldsymbol{u}_p \times \boldsymbol{B}_p) \cdot d\boldsymbol{S} = \int_{S_\varepsilon} \eta\, (\nabla \times \boldsymbol{B}_p) \cdot d\boldsymbol{S}. \tag{10}$$

If we apply Stokes' theorem [7] to the right side of (10), then

$$\int_{S_\varepsilon} (\boldsymbol{u}_p \times \boldsymbol{B}_p) \cdot d\boldsymbol{S} = \oint_{C_\varepsilon} \eta \boldsymbol{B}_p \cdot d\boldsymbol{x}. \tag{11}$$

Here, S_ε is the area inside the line integral on the right side of (11), and C_ε is the circle of the line integral. The approximate values on the left side of (11) are as follows:

$$\left| \int_{S_\varepsilon} (\boldsymbol{u}_p \times \boldsymbol{B}_p) \cdot d\boldsymbol{S} \right| \leq U B_\varepsilon S_\varepsilon. \tag{12}$$

Here, the average magnetic field in the line integration on the right side of (11) is B_ε. The area of each integral is the same. U is the maximum flow velocity in the area. Since the magnetic field is zero in the centre of the N-point, the mean magnetic field within S_ε is smaller than B_ε (whether the N-point is concave or convex; see Appendix B), but at (12), it is estimated to be B_ε. Then, the relationship between the magnitude of the values on the right side and the left side of (11) is $\eta_N \varepsilon B_\varepsilon \leq U B_\varepsilon S_\varepsilon$ or

$$\eta_N \varepsilon \leq U S_\varepsilon, \tag{13}$$

where ε on the left side is the perimeter of the N-point area. (To contrast (13) with (11), it may be easier to understand if the left and right sides are reversed, i.e., $US_\varepsilon \geq \eta_N \varepsilon$.) However, since $\varepsilon \to 0$ results in $S_\varepsilon \to \mathrm{O}(\varepsilon^2)$, (13) is not compatible for any finite values $\frac{U}{\eta_N}$. Therefore, it is conventionally thought that this magnetic field arrangement is contradictory, so an axisymmetric magnetic field does not occur.

However, for the following fundamental reasons, this paper refutes the N-point claim, which is unrelated to the claim in Section 2.1. First, the right side of (11) is the right side of (10) according to Stokes' theorem. That is, (11) is an area integral on the left and right sides. (11) shows that the left and right sides are always the same. Applying different order changes to the areas on the left and right sides is incorrect and misleading. Therefore, the N-point claim is invalid.

Examining the properties of the additional terms demonstrates that they can be a 'source' of power generation. Considering these additional terms, the problem was solved because the induction equation contains the mechanism by which axisymmetric magnetic fields are generated from axisymmetric convection. Thus, the convection and magnetic fields are axisymmetric, leading to simple magnetic field generation. In addition, it was separately demonstrated that the N-point claim was invalid.

3 Discussion

We have shown that an axisymmetric magnetic field can be generated by axisymmetric convection and that magnetic field generation occurs through simple poloidal convection. However, this demonstration does not fully explain the stable generation of a magnetic field. In addition, the deformation of the formula contains unresolved points. Here, the problems and supplements of this paper are described.

For the reasons shown in the previous section, related fields can be studied from the perspective of including the ideas of this paper in Cowling's theorem. However, (4) does not fully represent convection, which is difficult to model accurately. Convection behaviour is important for discussing the maintenance of a magnetic field, but this topic is beyond the scope of this paper. Therefore, whether convection further contributes to or interferes with the maintenance of the magnetic field is uncertain.

In one peer review, it was suggested to transform (5) via the following formula (hereinafter referred to as the deformation):

$$\nabla \times (A_\phi e_\phi) = \nabla \times (r A_\phi \nabla\phi) \tag{14}$$

where e_ϕ is the unit vector along ϕ, r is the cylindrical radius and $\nabla\phi$ is the gradient of the angle ϕ

$$e_\phi = r\nabla\phi \,, \psi = r A_\phi \tag{15}$$

$$\nabla \times (A_\phi e_\phi) = (\nabla\psi) \times (\nabla\phi) \tag{16}$$

There was an example of a website[10] that was transformed by a similar formula. Deformation is not used in this paper. It is considered but unresolved (see Appendix C), and this paper may not have sufficient evidence in this regard.

In the previous section, to explain the 'source' of magnetic field generation, Fig. 1 is used as an example of an asymmetric magnetic field arrangement bordered by the r-axis. Moreover, this figure is discussed as an example of the location and shape of N-points. This figure is a hypothetical illustration based on preprint data. Furthermore, this preprint discusses the growth magnetic field rather than the stable magnetic field. In other words, the argument part using this figure has weak evidence. However, there is at least a possibility that a source of power generation will emerge. This possibility exists because, as shown in Fig. 1, the flow velocity is reversed bordered by the r-axis, so the magnetic field is bent asymmetrically because of magnetic field freezing. Even if the magnetic field arrangement in Fig. 1 is inaccurate, this arrangement is asymmetrically bordered by the r-axis. Therefore, the vector potential of the magnetic field is also asymmetric, and the existence of the 'source' of power generation can be demonstrated.

Here, we describe the trend of power generation. In the previous formulas, such as (2) and (6), if the scale of these dimensions (r, ϕ, z) is arbitrarily changed, a similar result is obtained. Here, we consider the dimensions of the ring. Since (6) is a unit quantity in the ϕ direction, the circumference value is multiplied by $2\pi r$, so this term increases proportionally as the scale increases. In contrast, if $\boldsymbol{A}_\phi = \frac{\Phi \mathbf{e}_z}{2\pi r} \left(\Phi \mathbf{e}_z = \oint_C \boldsymbol{A}_\phi \cdot d\boldsymbol{l} = 2\pi r \boldsymbol{A}_\phi\right)$[7] is substituted into the decay term's $\boldsymbol{B}_p \,(= \nabla \times \boldsymbol{A}_\phi)$ of (2), the attenuation term decreases in inverse proportion to the increase in scale because the denominator r is multiplied by the electrical conductivity σ contained in $\eta\left(=\frac{1}{\sigma\mu}\right)$, and the resistance value decreases in inverse proportion. Here, $\boldsymbol{\Phi}\mathbf{e}_z$ is a magnetic flux linkage occurring inside the circumference C of the line integral. When combined, it becomes easier to generate electricity in proportion to the square of the scale increase. In contrast, if the scale is reduced, generating electricity becomes difficult. However, if the convection velocity is appropriate, electricity should be generated.

The information presented in the previous section is a novel concept. Combining the results of this study with conventional research methods in this field will expand the scope of research on fluid magnetic fields. A theory where axisymmetric magnetic fields are stably generated by simple convection may change the research conditions in related fields and lead to discoveries.

4 Conclusion

This paper presents a theory in which axisymmetric magnetic fields are stably generated by simple convection. We believe that this theory provides clues for related research. However, its application is limited; for example, it does not include calculations of convection behaviour. However, we demonstrate the possibility of axisymmetric magnetic fields on the basis of a novel concept, so we believe that this theory will be useful in certain situations. Even if the scale is small, electricity can be generated if the flow velocity is appropriate, as mentioned in the Discussion section. We believe that creating arbitrary convection is more feasible in an artificial plasma experimental facility than in the natural world, so the results of this research may be useful when aiming to strengthen magnetic fields. In this way, this paper has potential applications in the field of magnetohydrodynamics, and we expect that it will be useful not only for research on astronomical bodies but also for research conducted in plasma furnaces and sodium experimental facilities.

In any case, we believe that the results of this paper will contribute considerably to further elucidating the mechanism of magnetic field formation.

Declarations

Conflicts of interest.

The author has no conflicts of interest to declare.

Funding.

No funding was obtained for this work.

Data availability statement.

We do not analyze or generate any datasets, because our work proceeds via a theoretical and mathematical approach.

Appendix A Conventionally, No Source of Power Generation

Below, the total amount of energy is calculated for the part of (A1) excluding the additional terms of (6), which becomes zero:

$$-\left[\left(\frac{\partial P}{\partial r}\right)\frac{\partial A}{\partial z}-\frac{\partial P}{\partial z}\left(\frac{\partial A}{\partial r}\right)\right]e_{\phi}. \tag{A1}$$

(A1) is multiplied by A to obtain the energy and subjected to partial integration.

First, the first term of (A1), $-\frac{\partial P}{\partial r}\frac{\partial A}{\partial z}$, is multiplied by A and integrated over r as follows:

$$-\int \frac{\partial P}{\partial r}A\frac{\partial A}{\partial z}dr=-\left[PA\frac{\partial A}{\partial z}\right]+\int P\frac{\partial}{\partial r}\left(A\frac{\partial A}{\partial z}\right)dr=\int P\frac{\partial}{\partial r}\left(A\frac{\partial A}{\partial z}\right)dr. \tag{A2}$$

Here, P and A in the surface term are zero at $r=0$ and $r=\infty$. Since the integration over ϕ is uniform at the circumference, this equation is multiplied by $2\pi r$. Integrating it over z yields

$$\iiint -\frac{\partial P}{\partial r}A\frac{\partial A}{\partial z}dV=2\pi r\iint P\frac{\partial}{\partial r}\left(A\frac{\partial A}{\partial z}\right)drdz. \tag{A3}$$

Next, the second term of (A1), $\frac{\partial P}{\partial z}\frac{\partial A}{\partial r}$, is multiplied by A and integrated over z as follows:

$$\int \frac{\partial P}{\partial z}A\frac{\partial A}{\partial r}dz=\left[PA\frac{\partial A}{\partial r}\right]-\int P\frac{\partial}{\partial z}\left(A\frac{\partial A}{\partial r}\right)dz=-\int P\frac{\partial}{\partial z}\left(A\frac{\partial A}{\partial r}\right)dz. \tag{A4}$$

Here, P and A in the surface term are zero at $z=\pm\infty$. Since the integration over ϕ is uniform at the circumference, this equation is multiplied by $2\pi r$. Integrating it over r yields

$$\iiint \frac{\partial P}{\partial z}A\frac{\partial A}{\partial r}dV=-2\pi r\iint P\frac{\partial}{\partial z}\left(A\frac{\partial A}{\partial r}\right)drdz. \tag{A5}$$

Then, adding (A3) and (A5),

$$\begin{aligned}&2\pi r\iint P\frac{\partial}{\partial r}\left(A\frac{\partial A}{\partial z}\right)drdz-2\pi r\iint P\frac{\partial}{\partial z}\left(A\frac{\partial A}{\partial r}\right)drdz\\&=2\pi r\iint P\left[\frac{\partial}{\partial r}\left(A\frac{\partial A}{\partial z}\right)-\frac{\partial}{\partial z}\left(A\frac{\partial A}{\partial r}\right)\right]drdz.\end{aligned} \tag{A6}$$

The square brackets of (A6) are further decomposed as follows:

$$\frac{\partial}{\partial r}\left(A\frac{\partial A}{\partial z}\right)-\frac{\partial}{\partial z}\left(A\frac{\partial A}{\partial r}\right)=\frac{\partial A}{\partial r}\frac{\partial A}{\partial z}+A\frac{\partial}{\partial r}\left(\frac{\partial A}{\partial z}\right)-\frac{\partial A}{\partial z}\frac{\partial A}{\partial r}-A\frac{\partial}{\partial z}\left(\frac{\partial A}{\partial r}\right). \tag{A7}$$

The first and third terms on the right side of (A7) offset each other. Since partial derivatives do not depend on the order, the second and fourth terms also cancel each other. Therefore, in terms of power generation, (A1) becomes zero regardless of the poloidal position before integration.

Appendix B Whether the N-point is Concave or Convex

Please note that this explanation is supplementary to understanding conventional theory, not the plot of the argument of this paper.

The concavity or convexity of the N-point is considered. The results are the same in both cases, but the results of both methods are described here to aid in understanding Fig. 1. In conventional N-point theory, the magnetic field is assumed to be smaller than B_ε on average within S_ε, so the N-point is assumed to be concave. However, since the N-point, such as that in Fig. 1, is orbited by a magnetic field, a current must exist within that area. Therefore, considering the N-point as a convexity in which the magnetic field increases towards the centre is reasonable. Then, the magnetic field is larger than B_ε on average within S_ε. Therefore, in area integration, since the approximate estimation is made by B_ε, the direction of the inequality sign in (13) is $\eta_N \varepsilon > US_\varepsilon$. However, if the average magnetic field in S_ε changes by $\varepsilon \to 0$ and $S_\varepsilon \to \mathrm{O}(\varepsilon^2)$, the direction of the inequality sign in this equation does not change, but the ratio of the right and left sides changes. This magnetic field arrangement is not compatible with any finite values of $\frac{U}{\eta_N}$. Therefore, conventional theory argues that a poloidal magnetic field arrangement is impossible, regardless of whether the N-point is concave or convex.

Appendix C Notes on Transforming Formulas

The peer review suggestion and example website illustrate the deformation in (16). The question of this is expressed below.

C.1 Cartesian and Cylindrical

It is important not to lose minute components when calculating the components of $\mathrm{rot}\boldsymbol{P}$ and $\mathrm{rot}\boldsymbol{A}$. However, in the original text of Cowling's theorem, the components of convection and magnetic fields presented are missing minute components. Moreover, no method of decomposition into these components is shown. The problem lies in whether to decompose the components in Cartesian coordinates or cylindrical coordinates. The reason why a problem arises when decomposing $\mathrm{rot}\boldsymbol{P}$ and $\mathrm{rot}\boldsymbol{A}$ into components after applying the formula corresponding to equation $\nabla \times (A\mathbf{e}_\phi) \equiv \nabla A \times \mathbf{e}_\phi$ is explained as follows. The following is a comparison of the cases using Cartesian coordinates and cylindrical coordinates. Next, we explain why the components are decomposed directly via the mathematical formula of cylindrical coordinates without deformation.

The mathematical formulas for some of the vectors involved are shown below. In Cartesian coordinates (x, y, z), the rotation $\mathrm{rot}\,\boldsymbol{V}$ of the vector $\boldsymbol{V}$ is as follows:

$$\nabla \times \boldsymbol{V} = \left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right)_x + \left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right)_y + \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)_z. \tag{C8}$$

The subscripts at the bottom right of the symbols and parentheses indicate the components in these directions. In the following derivation, the vector $\boldsymbol{V}$ of $\nabla \times \boldsymbol{V}$ is separated and substituted into the scalar component f and the vector component $\boldsymbol{V}$ (later, it is treated as a fundamental vector). To observe the components of the equation, we derive (C10), which is the basis of the equation, from (C8). The x component is as follows:

$$\begin{aligned}(\nabla \times f\boldsymbol{V})_x &= \frac{\partial}{\partial y}(fV_z) - \frac{\partial}{\partial z}(fV_y) \\ &= \frac{\partial f}{\partial y}V_z + f\frac{\partial V_z}{\partial y} - \frac{\partial f}{\partial z}V_y - f\frac{\partial V_y}{\partial z} \\ &= \left(\frac{\partial f}{\partial y}V_z - \frac{\partial f}{\partial z}V_y\right) + f\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right).\end{aligned} \tag{C9}$$

The other components are the same as those in (C9), and (C10) is derived.

$$\nabla \times (f\boldsymbol{V}) = \nabla f \times \boldsymbol{V} + f\nabla \times \boldsymbol{V}. \tag{C10}$$

The following formula is commonly used in electromagnetism.

$$\nabla \times (\nabla p) = 0. \tag{C11}$$

If p is considered a scalar potential and $\boldsymbol{V}$ in (C10) is a vector of ∇p, the second term on the right side of (C10) becomes zero, as shown in (C11). Then, an equation equivalent to the formula used for formula deformation can be obtained as follows:

$$\begin{aligned}\nabla \times (f\nabla p) &= \nabla f \times \nabla p + f\nabla \times \nabla p, \\ &= \nabla f \times \nabla p.\end{aligned} \tag{C12}$$

For ease of understanding, the symbol $f\nabla p$ is changed to the vector potential $\boldsymbol{A} = A\mathbf{e}_\phi$ as follows:

$$\nabla \times (A\mathbf{e}_\phi) = \nabla A \times \mathbf{e}_\phi. \tag{C13}$$

The problem, however, is that (C13) is derived from (C8), which is an equation for Cartesian coordinates.

The component decomposition of rot $\boldsymbol{V}$ of the vector $\boldsymbol{V}$ by cylindrical coordinates (r, ϕ, z) is as follows:

$$\nabla \times \boldsymbol{V} = \left[\frac{1}{r}\left(\frac{\partial V_z}{\partial \phi} - r\frac{\partial V_\phi}{\partial z}\right)\right]_r + \left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}\right)_\phi + \left[\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_\phi) - \frac{\partial V_r}{\partial \phi}\right)\right]_z. \tag{C14}$$

The subscripts at the bottom right of the symbols and parentheses indicate the components in these directions. To observe the components of the equation, we derive equations from (C14). The r component is as follows:

$$
\begin{aligned}
(\nabla \times f\boldsymbol{V})_r &= \frac{1}{r}\left[\frac{\partial}{\partial\phi}(fV_z) - r\frac{\partial}{\partial z}(fV_\phi)\right] \\
&= \frac{1}{r}\left[\left(\frac{\partial f}{\partial\phi}V_z + f\frac{\partial V_z}{\partial\phi}\right) - r\left(\frac{\partial f}{\partial z}V_\phi + f\frac{\partial V_\phi}{\partial z}\right)\right] \\
&= \frac{1}{r}\frac{\partial f}{\partial\phi}V_z + \frac{1}{r}f\frac{\partial V_z}{\partial\phi} - \frac{\partial f}{\partial z}V_\phi - f\frac{\partial V_\phi}{\partial z} \\
&= \frac{1}{r}\frac{\partial f}{\partial\phi}V_z - \frac{\partial f}{\partial z}V_\phi + f\left(\frac{1}{r}\frac{\partial V_z}{\partial\phi} - \frac{\partial V_\phi}{\partial z}\right)
\end{aligned}
$$

If ∇ is $\nabla' = (\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial\phi}, \frac{\partial}{\partial z})$, the r component is as follows:

$$
(\nabla \times f\boldsymbol{V})_r = (\nabla' f \times \boldsymbol{V} + f\nabla' \times \boldsymbol{V})_r\,. \tag{C15}
$$

The ϕ component is as follows:

$$
\begin{aligned}
(\nabla \times f\boldsymbol{V})_\phi &= \frac{\partial}{\partial z}(fV_r) + \frac{\partial}{\partial r}(fV_z) \\
&= \frac{\partial f}{\partial z}V_r + f\frac{\partial V_r}{\partial z} - \frac{\partial f}{\partial r}V_z - f\frac{\partial V_z}{\partial r} \\
&= \left(\frac{\partial f}{\partial z}V_r - \frac{\partial f}{\partial r}V_z\right) + f\left(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}\right)
\end{aligned}
$$

This result means the follwing:

$$
(\nabla \times f\boldsymbol{V})_\phi = (\nabla f \times \boldsymbol{V} + f\nabla \times \boldsymbol{V})_\phi\,. \tag{C16}
$$

The z component is as follows:

$$
\begin{aligned}
(\nabla \times f\boldsymbol{V})_z &= \frac{1}{r}\frac{\partial}{\partial r}(rfV_\phi) - \frac{\partial}{\partial\phi}(fV_r) \\
&= \frac{1}{r}\left[\frac{\partial r}{\partial r}fV_\phi + r\frac{\partial}{\partial r}(fV_\phi)\right] - \left(\frac{\partial f}{\partial\phi}V_r + f\frac{\partial V_r}{\partial\phi}\right) \\
&= \frac{1}{r}\left[fV_\phi + r\left(\frac{\partial f}{\partial r}V_\phi + f\frac{\partial V_\phi}{\partial r}\right)\right] - \left(\frac{\partial f}{\partial\phi}V_r + f\frac{\partial V_r}{\partial\phi}\right) \\
&= \frac{1}{r}fV_\phi + \frac{\partial f}{\partial r}V_\phi + f\frac{\partial V_\phi}{\partial r} - \frac{\partial f}{\partial\phi}V_r - f\frac{\partial V_r}{\partial\phi} \\
&= \frac{1}{r}fV_\phi + \left(\frac{\partial f}{\partial r}V_\phi - \frac{\partial f}{\partial\phi}V_r\right) + f\left(\frac{\partial V_\phi}{\partial r} - \frac{\partial V_r}{\partial\phi}\right)
\end{aligned}
$$

This result means the following:

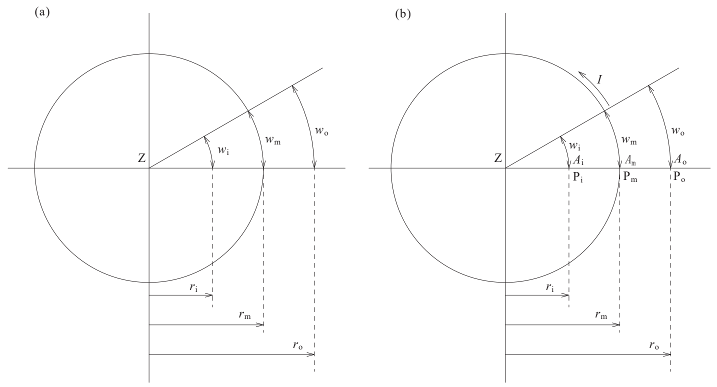

Fig. C1 Diagram of cylindrical convection and vector potential
(a) Top view of convection. (b) Top view of the vector potential

$$(\nabla \times f\boldsymbol{V})_z = \left(\frac{1}{r} f V_\phi + \nabla f \times \boldsymbol{V} + f \nabla \times \boldsymbol{V}\right)_z . \tag{C17}$$

As described above, owing derivation by cylindrical coordinates, (C15) and (C17) differ from the case of Cartesian coordinates because a term containing the coefficient $\frac{1}{r}$ appears in several places. Therefore, the deformation of the equation according to (C13), which is based on Cartesian coordinates, cannot be used for examination in cylindrical coordinates. To be clear, substituting (15) within the right-hand bracket of (14) is not a problem. There is no problem if the rot on both sides of (14) and the rot on the left side of (16) are treated in cylindrical coordinates. The problem arises when the transformation of the right side of (14) into the right side of (16), (C12) or (C13), which holds in Cartesian coordinates, is used. First, since (C14) strictly decomposes the components in cylindrical coordinates, it is not considered necessary to transform the equation according to (C13).

The conclusion of this paper depends on whether the deformation is used. However, we have not found literature that shows why or how deformation is used. Therefore, we cannot confirm at this time whether it is simply a convention, who proposed it, what is a valid reason, or whether Cowling was aware of the deformation. Future investigations are needed.

C.2 Illustration via Diagram

Additional terms for the axisymmetric flow velocity and magnetic field are illustrated via diagrams to facilitate geometric understanding.

Observe Fig. C1. First, convection is described. (a) Top view from the top of axisymmetric axis Z. The arrows indicated by w_i, w_m and w_o are the inner, middle and outer channel widths, respectively. Consider the portion distributed at the angle indicated by the arrow. The arrows indicated by r_i, r_m and r_o are the distances from the Z-axis to the inner, middle and outer channels, respectively. The circle is a line indicating the distance of r_m from the Z-axis. The width of the flow path thus varies in cylindrical coordinates in proportion to the distance from the Z-axis. In other words, the width of the flow path is increased by r. Then, for a certain amount of flow to pass through these widths, flow needs to pass at a flow velocity of $\frac{1}{r}$. However, the flow velocity is determined by the setting of convection. The additional term is $\frac{1}{r}P$ when P is constant within the range of settings, which is not always true in reality. However, even if P is not constant, an additional term replaces $\frac{1}{r}P$. Normally, P is a function of r and z.

Next, the vector potential is described. (b) is a top view as above. Suppose that a current I flows along the circle. P_i and P_o indicate some positions inside and outside the current path, resulting in vector potentials of A_i and A_o, respectively. Consider the portion distributed at the angle indicated by the arrow. r_i, r_m and r_o are the same as above. The same is true for magnetic fields. If the length of the current source is w_m, the vector potentials A_i and A_o (within widths w_i, and w_o) are received from the current for that length according to the distance from P_m. In addition, it is distributed according to the widths w_i and w_o. However, if there is only one current path, as shown in the figure, the additional term will not be $\frac{1}{r}A$. The additional term is $\frac{1}{r}A$ when A is constant within the range of settings, which is not always true in reality. However, even if A is not constant, an additional term replaces $\frac{1}{r}A$. Normally, A is a function of r and z.

If (C13) is used, the terms $\frac{1}{r}A$ and $\frac{1}{r}P$ do not occur, so the flow velocity and magnetic field equations are as follows:

$$\boldsymbol{u}_p = -\frac{\partial P}{\partial z}\mathbf{e}_r + \frac{\partial P}{\partial r}\mathbf{e}_z. \tag{C18}$$

$$\boldsymbol{B}_p = -\frac{\partial A}{\partial z}\mathbf{e}_r + \frac{\partial A}{\partial r}\mathbf{e}_z. \tag{C19}$$

Then the axisymmetric feature disappears. Even if the width of the flow path or the length of the electric circuit differs from w_i to w_o, the flow velocity and magnetic field are irrelevant to these widths or lengths. Therefore, since the property of axisymmetry is excluded, the deformation will lead to a different result from the phenomenon of axisymmetry. However, we think that the deformation is mentioned in a peer review and on the website for a reason, but we have not been able to find this reason.

Additionally, if power is generated, the current is more complicated because it is generated by a plurality of circles. To obtain the value accurately, numerical calculations must be relied on, which is beyond the scope of this paper. However, at least from the viewpoint of cylindrical coordinates, an additional term is generated to replace the term $\frac{1}{r}A$, and this term is considered a source of power generation. We consider that the conclusion is different from that of using (C13), which is based on Cartesian coordinates.

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