Axisymmetric Magnetic Fields can be Stably Generated by Simple Convection

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Researchers have struggled to understand the mechanism underlying the formation of celestial magnetic fields. The concept that complex convection is necessary to generate a magnetic field in some studies, including Cowling's theorem, constrains research. Research in this field could progress through the discovery of a simple mechanism. This paper addresses simple axisymmetric poloidal convection and magnetic fields. The problem arises when rigorously deriving the electromagnetic induction equations. This is explained by comparison using Cowling's theorem as an example. Cowling's theory omits certain elements from this equation. However, it was found that these elements are the very essence of axisymmetric magnetic field generation. In other words, the meaning of Cowling's theorem is reversed. This is a novel concept. As a result, even under simple axisymmetric convection, the possibility of generating axisymmetric magnetic fields is demonstrated. These findings will contribute to further elucidating the mechanism of magnetic field formation.

Keywords: Cowling's theorem, dynamo theory, simple convection, celestial magnetic fields

1. Introduction Researchers have long struggled to understand the mechanism underlying the formation of celestial magnetic fields. The famous foundations for elucidating the mechanism of the formation of celestial magnetic fields are the ω effect [1], the α effect [2], and Cowling's theorem [3].

Taking the Sun as an example, the magnetic field in the plane perpendicular to the axis of rotation of the Sun is called the toroidal magnetic field, and the magnetic field in the plane parallel to the axis of rotation is called the poloidal magnetic field. The same is true for convection.

According to Cowling's theorem, axisymmetric convection does not generate a stable axisymmetric magnetic field, either poloidal or toroidal.

The ω effect generates a toroidal magnetic field from a poloidal magnetic field where there is a gradient in angular velocity. Since the rotation of the surface of the Sun is faster at the equator than at the poles, there is an angular velocity gradient. If there is a poloidal magnetic field as the initial magnetic field, the magnetic field is stretched so that it is wound up by the angular velocity gradient, and the poloidal magnetic field becomes a toroidal magnetic field. If the toroidal magnetic field is changed to a poloidal magnetic field, the magnetic field may be amplified. However, no such effect has been found. In the end, the results support Cowling's theorem.

The α effect assumes a velocity field that twists a magnetic field. The concept is to twist the toroidal magnetic field in some places and direct it in the poloidal direction. Therefore, if an α effect is added to the ω effect, a mutual exchange of magnetic fields is possible, and the magnetic field can be amplified. However, this approach is not as easy to use as described above. Researchers are combining these effects with complex convection to further elucidate the mechanism of magnetic field generation.

The notion that a magnetic field is generated by complex convection or that an axisymmetric magnetic field does not occur constrains research. If it is clarified that a magnetic field can be generated by simpler convection, research in this field will further advance. This paper explores the possibility of generating a magnetic field by convection, which is a simpler concept.

We compare our theory with Cowling's theorem, which is the most famous and most effective theorem for explaining our theory. The problem arises by rigorously deriving the electromagnetic induction equations. Cowling's theory omits certain elements from this equation. However, it has been found that these elements are the very essence of axisymmetric magnetic field generation. In other words, the meaning of Cowling's theorem is reversed. This is a novel concept. To illustrate the differences, the original text of Cowling's theorem is shown and compared. The problem part of the equation is the induction term. If we derive the induction term strictly, we find two terms that are not in the equation of Cowling's theory. The meaning of these terms (corresponding to self-excited power generation) and their properties will be described. Moreover, there are factors that contribute more to power generation in these two terms. However, since it is difficult to explain these factors in detail in this paper, only the concept is described.

However, the above does not fully explain the stable generation of the magnetic field. The process and concerns of the stability of the magnetic field are addressed in the Discussion section. There is a lack of evidence for stable magnetic fields; however, we believe that the above is a useful theory, so we describe our expectation in the Conclusion section.

2. Description of the Problem This paper explores the possibility of generating a magnetic field by convection, which is a simpler approach. The differences between the findings and the conventional theories are shown, and the points elucidated in this study are explained below.

Unless otherwise stated, symbols or similar symbols with the same meaning as those used in Cowling's paper were used here; these meanings were transcribed almost verbatim in " ". Where there is no explanation, we provide a general interpretation.

"Let ρ denote the density of the gas, and $\boldsymbol{c}$ denote its velocity (or speed) at any point; also, let $\boldsymbol{H}$ be the magnetic intensity."

The electromagnetic induction equation used in Cowling's theory is derived from Ohm's law as follows:

$$\boldsymbol{j} = \sigma(\boldsymbol{c}_\wedge \boldsymbol{H} - grad\, V) \tag{1}$$

Here, "the electric force on the gas due to its motion in a magnetic field is, in E.M.U., given by $\boldsymbol{c}_\wedge \boldsymbol{H}$; the electrostatic force is $-grad\, V$, where V is the electrostatic potential, which we also assume is measured in E.M.U. Hence, if $\boldsymbol{j}$ is the electric current density and σ is the conductivity of the gas", the operator $\wedge$ is not explained in Cowling's paper. It must be a wedge product. Hereinafter, the wedge product is treated as the cross product.

Since this theorem discusses whether the stability of the magnetic field occurs electromagnetically, the electromotive force [4] due to fluctuations in the vector potential is considered omitted. For the sake of explanation, a section on the electromotive force has been added, as shown in Ohm's law below.

$$\boldsymbol{j} = \sigma\left(\boldsymbol{c}_\wedge \boldsymbol{H} - \frac{\partial \boldsymbol{A}}{\partial t} - grad\, V\right) \tag{2}$$

"Let Oz be taken as the axis of symmetry, and let ϖ denote the distance of any point from this axis, so that $\varpi^2 = x^2 + y^2$." We then interpret x, y and z to mean the values of the Cartesian coordinates and their directions. By using the terms derived in the paper by Cowling, Eq. (2) can be reorganized into an electromagnetic induction equation as follows:

$$\frac{\partial \boldsymbol{A}}{\partial t} = \left[\frac{1}{4\pi\varpi\sigma}\left(\frac{\partial^2\psi}{\partial\varpi^2} + \frac{\partial^2\psi}{\partial z^2} - \frac{1}{\varpi}\frac{\partial\psi}{\partial\varpi}\right) - \frac{1}{\rho\varpi^2}\left(\frac{\partial\phi}{\partial\varpi}\frac{\partial\psi}{\partial z} - \frac{\partial\phi}{\partial z}\frac{\partial\psi}{\partial\varpi}\right)\right]_{\phi'} \tag{3}$$

Note that Eq. (3) is only the ϕ' component that rotates Oz. Here, "there exists a generalized Stokes stream function ϕ, depending on z and ϖ, such that the components of $\boldsymbol{c}$ are parallel to and perpendicular to Oz. ψ is a function of ϖ and z, analogous to the Stokes function; the total magnetic induction across an area perpendicular to Oz bounded by a circle centered on Oz, which passes through a given point, is equal to the value of $2\pi\psi$ at that point." Therefore, the term is added to the left side of the electromagnetic induction equation. According to Cowling's theory, this left side is considered zero.

If we substitute pole conditions ($\frac{\partial\psi}{\partial z} = 0$, $\frac{\partial\psi}{\partial\varpi} = 0$ and $\frac{\partial^2\psi}{\partial\varpi^2} + \frac{\partial^2\psi}{\partial z^2} \neq 0$) into Eq. (3), the second term on the right side becomes zero; however, the first term is not zero. Therefore, the right side is not zero, and the left side, which is zero and leads to contradiction in Cowling's paper, fluctuates. Thus, even if there is a specific pole with a nonzero second-order partial derivative, Eq. (3) is satisfied.

The first term on the right side of Eq. (3) is referred to as the attenuation term, and the second term is referred to as the induction term. Because the induction term is zero, this equation can be interpreted as indicating that the magnetic flux decreases only due to attenuation according to the attenuation term.

However, the generation of the magnetic field can be explained by two terms that appear by strictly treating the derivation of the induction term of the electromagnetic induction equation. Cowling's paper omits these terms. The exact derivation is shown and explained below.

Strictly speaking, the induction term does not become zero even if the conditions of the pole are given. In fact, there is a part that is omitted in the process of deriving the induction term. To illustrate this, $\boldsymbol{c}_\wedge\boldsymbol{H}$ is calculated, including the omitted part. Since it is calculated in cylindrical coordinates, $\boldsymbol{c}$ and $\boldsymbol{H}$ are expressed as follows:

$$\rho\boldsymbol{c} = \nabla_\wedge \frac{1}{\varpi}(\phi)_{\phi'} = \frac{1}{\varpi}\left[\left(-\frac{\partial\phi}{\partial z}\right)_\varpi + \left(\frac{1}{\varpi}\frac{\partial}{\partial\varpi}(\varpi\phi)\right)_z\right]$$

$$= \frac{1}{\varpi}\left[\left(-\frac{\partial\phi}{\partial z}\right)_\varpi + \left(\frac{1}{\varpi}\phi + \frac{\partial\phi}{\partial\varpi}\right)_z\right] \tag{4}$$

$$\boldsymbol{H} = \nabla_\wedge \frac{1}{\varpi}(\psi)_{\phi'} = \frac{1}{\varpi}\left[\left(-\frac{\partial\psi}{\partial z}\right)_\varpi + \left(\frac{1}{\varpi}\frac{\partial}{\partial\varpi}(\varpi\psi)\right)_z\right]$$

$$= \frac{1}{\varpi}\left[\left(-\frac{\partial\psi}{\partial z}\right)_\varpi + \left(\frac{1}{\varpi}\psi + \frac{\partial\psi}{\partial\varpi}\right)_z\right] \tag{5}$$

Here, the subscripts at the bottom right of each parenthesis in the curly braces indicate the ϖ-axis component and the z-axis component, respectively. Since $\frac{1}{\varpi}$

decreases when it is sufficiently far from Oz, we note that $\frac{1}{\varpi}\phi$ and $\frac{1}{\varpi}\psi$ are omitted in Eq. (3). By using Eqs. (4) and (5), the formula is as follows:

$$\rho \boldsymbol{c}_{\wedge} \boldsymbol{H} = \frac{1}{\varpi}\left[\left(-\frac{\partial\phi}{\partial z}\right)_{\varpi} + \left(\frac{1}{\varpi}\phi + \frac{\partial\phi}{\partial\varpi}\right)_{z}\right]_{\wedge} \frac{1}{\varpi}\left[\left(-\frac{\partial\psi}{\partial z}\right)_{\varpi} + \left(\frac{1}{\varpi}\psi + \frac{\partial\psi}{\partial\varpi}\right)_{z}\right] \quad (6)$$

Eq. (6) is calculated as follows:

$$\boldsymbol{c}_{\wedge} \boldsymbol{H} = -\frac{1}{\rho}\frac{1}{\varpi^2}\left[\left(\frac{1}{\varpi}\phi + \frac{\partial\phi}{\partial\varpi}\right)_{z}\left(\frac{\partial\psi}{\partial z}\right)_{\varpi} - \left(\frac{\partial\phi}{\partial z}\right)_{\varpi}\left(\frac{1}{\varpi}\psi + \frac{\partial\psi}{\partial\varpi}\right)_{z}\right]_{\phi'} \quad (7)$$

Note that Eq. (7) is only the ϕ' component that rotates Oz. If the second term on the right-hand side of Eq. (3) is replaced with Eq. (7), the additional terms $\frac{1}{\varpi}\phi$ and $\frac{1}{\varpi}\psi$ are added. Even if the conditions of the poles are substituted into this equation, the induction term does not become zero due to this extra term.

The functions and properties of the added terms are described below.

From Eq. (7), the part related to term $\frac{1}{\varpi}\phi$ is extracted and transformed as follows:

$$-\frac{1}{\rho}\frac{1}{\varpi^2}\left(\frac{1}{\varpi}\phi\right)_{z}\left(\frac{\partial\psi}{\partial z}\right)_{\varpi} = -\frac{1}{\rho}\frac{1}{\varpi^2}\frac{1}{\varpi}\rho\left(f^{-1}\left(\boldsymbol{c}\right)\right)_{z}\left(\frac{\partial\psi}{\partial z}\right)_{\varpi}$$
$$= -\frac{1}{\varpi^2}\frac{1}{\varpi}\left(\frac{\partial\psi}{\partial t}\right)_{\phi'} = -\frac{1}{\varpi^2}\frac{\partial\boldsymbol{A}}{\partial t}. \quad (8)$$

Here, $\frac{1}{\varpi}\left(\frac{\partial\psi}{\partial t}\right)_{\phi'} = \frac{\partial\boldsymbol{A}}{\partial t}$, and the inverse function of Eq. (4) is $\phi = f^{-1}\left(\rho\boldsymbol{c}\right)$. This inverse function is not accurate as an equation because it does not always determine one ϕ from $\rho\boldsymbol{c}$, but in real convection, it is considered possible to separate them into a one-to-one relationship by setting each condition. Furthermore, in general, the function $f^{-1}\left(\rho\boldsymbol{c}\right)$ is not $\rho f^{-1}\left(\boldsymbol{c}\right)$, but among the various convections, we believe that there is a convection that establishes this. Therefore, even if $\frac{\partial\psi}{\partial z}$ is zero, a time-varying term, $\frac{\partial\boldsymbol{A}}{\partial t}$, may occur. If this term is added to Eq. (3), there is a possibility that the value exceeds the attenuation term depending on the condition. For example, a change in the vector potential $\boldsymbol{A}$ also occurs in a mechanical structural change in a conductive fluid. Since it becomes a positive value depending on the direction of change, the left side of Eq. (3) is increased more to generate electricity. The magnetic field also increases due to its power generation. In other words, this term corresponds to self-excited power generation. Here, it is referred to as the main self-excitation power generation. This term is multiplied by $\frac{1}{\varpi}$ compared to the attenuation term. Therefore, since the value decreases as it moves away from Oz, the main position of power generation is considered close to Oz.

In addition, $\frac{1}{\varpi}\psi$ is as follows:

$$\frac{1}{\rho}\frac{1}{\varpi^2}\left(\frac{\partial\phi}{\partial z}\right)_{\varpi}\left(\frac{1}{\varpi}\psi\right)_{z} = \frac{1}{\rho}\frac{1}{\varpi^2}\left(\frac{\partial\phi}{\partial z}\frac{1}{\varpi}\psi\right)_{\phi'}. \quad (9)$$

This term increases depending on the z-gradient of the stream function and the strength of the magnetic field. If this term is added to Eq. (3), power generation may increase. If the magnetic field increases due to the main self-excitation power generation, it acts as if to encourage it. This process is referred to as sub-self-excitation power generation. However, if the magnetic field and the stream function weaken as they move away from Oz, the forces of this term are synergistically weak.

Since these terms are added to Eq. (3), the equation may be established even if the left side is zero. In other words, the meaning of Cowling's theorem is reversed.

In addition, there are other items on the side of power generation. In the above description, the equation is given only for a single place where the possibility of power generation is highest. Here, a single place is a position on a plane including Oz. However, since electromagnetic induction occurs at the same time at other positions (especially near the position where the potential for power generation is maximal), it is possible that electromagnetic interactions at those positions promote power generation. However, the interrelationship between them is beyond the scope of this paper.

The electromagnetic induction equation shown above is applied to a position on a plane including Oz, and there is no change in the direction around Oz. In other words, the added term also implies an axisymmetric phenomenon. Thus, convection and magnetic fields are axisymmetric, and simple magnetic field generation occurs.

3. Discussion We have shown that it is possible to generate an axisymmetric magnetic field from axisymmetric convection. Moreover, magnetic field generation occurs by simple poloidal convection. However, the above does not fully explain the stable generation of the magnetic field. Here, the process and concerns for the stability of the magnetic field are discussed.

Due to the power generation attributable to these added terms, the right side of Eq. (3) rises once (that is, the left side rises). Then, as the current increases, the attenuation term increases. As a result, the induction and attenuation terms cancel each other, and the right side is asymptotic to zero. In the end, even if the left side is zero, the magnetic field and attenuation are maintained at a certain strength, and Eq. (3) is established. Therefore, if we discuss only the above formula, the meaning of Cowling's theorem will be reversed. However, Eq. (4) does not fully represent convection, and it is difficult to do so. The behavior of convection is important for discussing the stability of the magnetic field, but this is beyond the scope of this paper. Therefore, it is not known whether convection further contributes to or interferes with the stability of the magnetic field.

In the above, we compare Cowling's theorem as an example; however, there are other anti-dynamo theorems, such as [5]. Verification of these issues is also necessary, but this is beyond the scope of this paper. A theory that stably generates axisymmetric magnetic fields with simple convection may change the research conditions in related fields and lead to new research results.

4. Conclusion This paper argues for a theory that stably generates axisymmetric magnetic fields with simple convection. We believe that this will provide clues for related research. However, there are limitations to its application; for example, it does not include calculations of convection behavior. However, at the very least, we are able to demonstrate the possibility of growing axisymmetric magnetic fields with a novel concept, so we believe that there are some useful situations. For example, there may be cases where the purpose is to grow a magnetic field. We believe that it is possible to create arbitrary convection to some extent in an artificial plasma experimental facility rather than in the natural world, so the results of this research may be useful when aiming to strengthen magnetic fields. In this way, this paper has potential application in so-called magnetohydrodynamics, and we expect that it will be useful not only for astronomical bodies but also for research in plasma furnaces and sodium experimental facilities.

In any case, we believe that the results of this paper will be of great help in further elucidating the mechanism of magnetic field formation.

Conflicts of interest.

The author has no conflicts of interest to declare.

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