Physical processes of mirror reversal based on the real image of the optical system

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Abstract: There has yet to be an established theory on the mirror problem, although various theories insist on the final resolution. The top reason is that this phenomenon is relevant to physical and cognitive processes. In previous works, the author proposed a comprehensive theory of the problem based on the concept of isotropic and anisotropic spaces. This time, the physical process was analyzed based on the real image formed in a camera system, utilizing the Cartesian coordinate system to successfully prove the mirror reversal mechanism. That may suggest the risk of using virtual images in analyzing problems relating to the visual image and more.

1. Introduction

1.1 Optics and mirror-reversal phenomenon

This study is relevant to the mirror problem, often expressed as why a mirror reverses right-left but not top-bottom and front-back. Nowadays, it may seem self-evident for some people that the specular reflection of the light on the mirror surface should be the decisive physical cause of the mirror reversal. However, any answer for the cause of the mirror reversal thus far has not been deduced directly from the reflection law. Optics (geometric optics) has been studied since the ancient Greek era, and it was in the 17th century that Huygens established the specular reflection principle, but it is said that there has been no established theory on the mirror problem to now. Thus, it may be no surprise that some scholars, particularly in psychological fields, including Gregory (1987) [6], Takano (1998, 2013) [12, 13], Bianchi, Ivana & Savardi, Ugo (2008) [2], and other recent researches, insist that optics is irrelevant to the mirror reversal phenomenon at least in some instances, or are ignoring optical conditions. For example, Takano (1998) defines a specific condition of mirror reversal as the "optical transformation," but he never verified or explained the optical process or cause of that specific condition until now. Most recent scientists have been explaining the problem geometrically using the enantiomorphism of the mirroring pair, which can be associated with mirror reflection. However, enantiomorphism alone can explain only the universal conditions for the problem and cannot explain specific cases and conditions. The connection between enantiomorphism and optical mechanism can be made only through the virtual image. The virtual image is not a physical existence, so the physical process cannot be analyzed through it. Physical processes concerning the optical image should have been analyzed based on the real image instead of the virtual image, though literature before my previous work had never employed the real image so long as it was found.

2. Methodological problems in previous studies and the aim of this study

2.1 The defect of methodologies seen in measure papers before

The problem has often been expressed verbally as a question of why a mirror reverses right-left but not top-bottom and front-back. Such a question has some anomalies because one can recognize top-bottom or front-back reversal depending on different conditions. Thus, there has been another defect or want in previous studies on the mirror problem. The want is the proper understanding of the meaning or notion of directional words such as top-bottom, front-back, and right-left, which are relevant to perceptual space but irrelevant to geometric space, which is relevant to the physical process. In the author's previous works [15, 16], the concept of isotropic space and anisotropic space, defined by E. Mach and E. Cassirer, has proved to be effective in facing such a challenge above. I dealt with this problem as a problem that consists of physical and cognitive processes. I have introduced the M-D-Rotation (Angle, Axis, or Plane) mechanism as the physical process. The mechanism was based on a diagram of the optical system, which included the mirror, object, and eyeball of the observer of the mirroring pair, and the analysis was made using both real and virtual images. In that optical system, the principle of specular reflection was expressed by figural illustrations, but the verbal explanation of the M-D-Rotation did not deduce the conclusion directly from the principle of specular reflection. That is why I reconsidered the M-D-Rotation mechanism to give it more verbal clarity and deep understanding.

From the above respect, this study has two aims as follows:

1. To clarify the physical factor in the mirror reversal phenomenon apart from the mental and cognitive factors.
2. To prove that Cartesian coordinates can be effectively applied to the physical mechanism but could not be effectively applied to virtual or suppositional existences in studying cognitive science because the Cartesian coordinate system is relevant to the isotropic space, so it would not be effective for the anisotropic space.

This study will clarify verbally the role of the specular reflection as the physical cause of the mirror reversal.

2.2 Limitations of Haig's (1993) argument

Haig (1993) [7] tried to explain the mirror reversal mechanism directly from the principle of specular reflection. He concluded the result: "To summarise, if the axis being assessed is perpendicular to the line between the observer and subject, it is classed as tangential. If the axis being assessed passes through the line between observer and subject, it is classed as sagittal. We may thus generalise these results by saying that viewing the tangential axis of an object in a plane mirror does not invert the image, whereas viewing the sagittal axis must result in a reversion of the image." He deduced this conclusion based on the analysis of physical rays. However, there is a breakdown between the argument of the ray's reflection and the reversion of the image because the ray's reflection is a physical phenomenon, whereas any image is something to be perceived in the human mind. It is essential to understand that the mirror problem is a problem concerning the image (visual image). The difference is that any direction in physical phenomena can defined only relatively, whereas any direction of the visual image should be defined by notions of top-bottom, front-back, and right-left. Haig (1993) defined top-bottom and right-left arbitrarily without any foundation. In other words, he has no idea about the meaning of top-bottom and right-left. So, Haig (1993) could not succeed in explaining the theory systematically in relation to the set of top-bottom, front-back, and right-left directions.
Furthermore, Haig (1993) defined the mirror problem as "The question of why a mirror reverses left and right, but not up and down" in the abstract. In this definition, the element front-back is missing. He cited this definition from Ittelson (1991), but Itteleson (19991) uses this expression as a "popular counterpart" of the 'mirror question', and Itteleson (19991) defines this problem as a 'mirror question,' and treats this problem three-dimensionally, including front-back. That means Haig (1993) analyzed the problem only two-dimensionally, which is a fatal error.

3. The analysis of this study

3.1 The concept of this analysis using the Cartesian coordinates

What proved is that any mirror reversal cannot be analyzed physically without the set of a convex lens and an image plane, and the object must be treated three-dimensionally. And when analyzed physically, the three-dimensional space is isotropic, so we can use the Cartesian coordinates to analyze. See Fig 1 and Fig 2 below. Fig 1 shows the situation in which the observer's eye perceives the mirror image of Pole A and B. Fig 2 illustrates the main physical elements of the above situation in which Cartesian coordinate is applied to the image plane and optical axis of the convex lens.

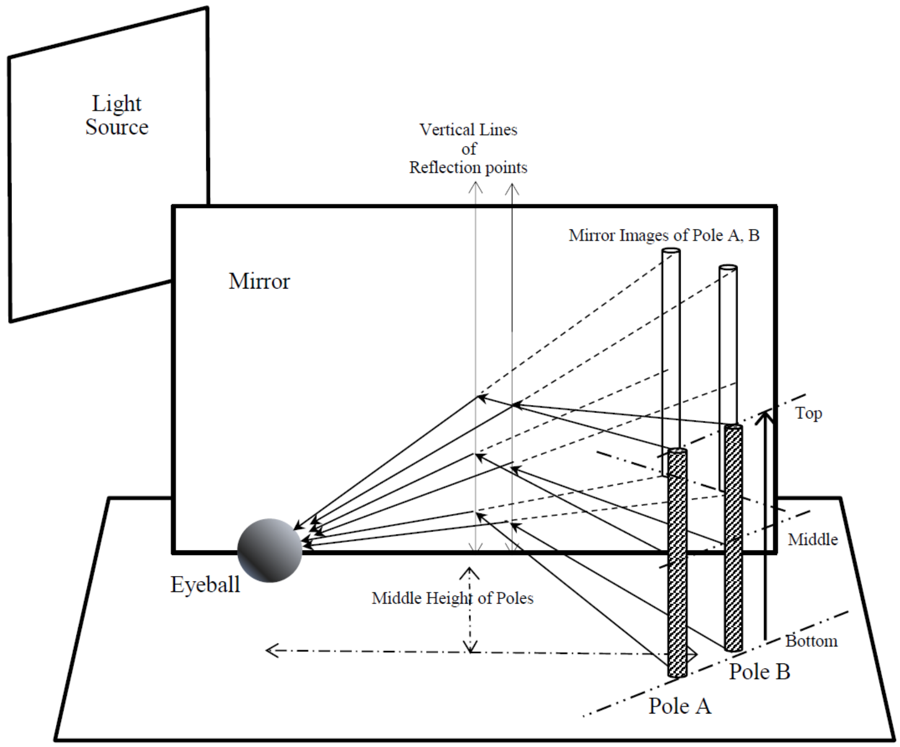

Figure 1 Schematic diagram of the analysis.

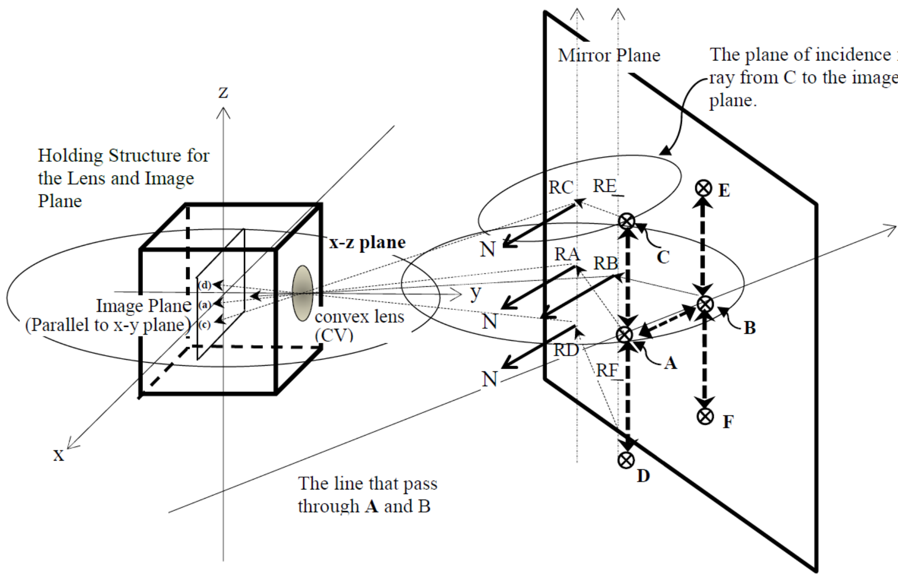

Figure 2 Applying the Cartesian coordinates to the system outlined in Figure 1.

In Fig. 2, the conditions are as follows:

1. The y-axis represents the optical axis of the convex lens.
2. The image plane is in the xz-plane because it is perpendicular to the y-axis, which is the optical axis.
3. The z-axis is parallel to the mirror surface so that it is perpendicular to the Normal line for any reflection point and can only translate and cannot turn or reverse so that its direction cannot change.
4. Light spots A, C, and D are on Pole A, Light spot B is on Pole B, and Light spots A and B are in the xy-plane.

There are some preconditions to notice:

1) Each of the arrows that represent a ray from each of light spots C, A, B, and D should be thought of as the principal ray of collective rays that pass through the lens and focus on the image plane as the image of the light point from each light spot.
2) Consequently, the endpoint of arrows that represent rays on the image plane should represent the formed image of each light spot.
3) In any context relevant to the Cartesian coordinate system and optics, the simple description of "image" means "real image," so it does not mean "virtual image."

Now, on the above conditions, we shall analyze the relative position for the real image of light spots formed on the image plane both in the line parallel to the z-axis and in the line parallel to the x-axis between when the ray comes via the mirror reflection and when the ray comes directly from the light spots. The y-axis is not relevant.

The analysis was done by dividing the situation into two conditions. One is when the mirror and image plane are parallel, and the other is when they are not.

3.2 Analyzing parallel to the z-axis

In the above conditions, RC, RA, and RD represent the reflection points of rays emitted from C, A, and D and focus on the image plane through the lens, respectively. At any reflection point, the Normal line of specular reflection is perpendicular to the mirror plane. It is perpendicular to any line on the plane and is included in any plane of incidence of the reflection point. In this situation, let us compare z-values for each reflection point. The possible z-values are given in the Table 1 below.

Table 1 Comparing relative z-values for the original light spots and the reflection points.

Original Light Spot	Reflection Point	Possible z-Value
C	RC	0 <
A	RA	0
D	RD	0 >

If the reflection point of C should be RA, the ray could not reach the optical center of the lens (z = 0) because the Normal line for RA is in the xy-plane so that the ray reaches any point below the xy-plane (z < 0) and for the reflection point of D, vice versa. Thus, the formed image of the light spots on the image plane reverses in a direction from the original order, which is the same as for rays emitted from those light spots and reach the image plane directly without mirror reflection. Figure 3 below shows the explanation above from a projection view on the yz-plane.

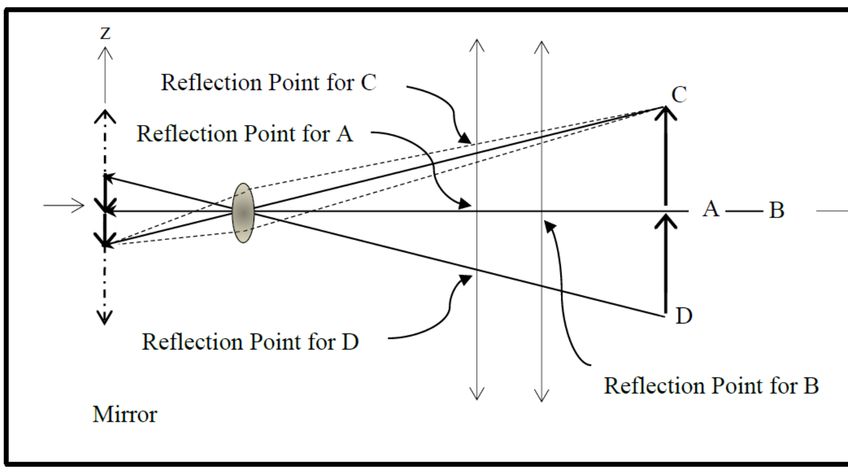

Figure 3 A projection view on the yz-plane of the optical diagram.

3.3 The difference between the z- and x-axes

When analyzing the line parallel to the z-axis as above, light spots C, A, and D are all in a line parallel to the z-axis, so their z-values are not relevant to the x-value and y-value for them. Therefore, when the coordinate system must move or rotate, which implies that the camera system must move and turn around the z-axis to take the image of C, A, and D directly without mirror reflection, the relative positions of their real images on the image plane of the camera cannot vary by the move or turning, so there is no need to consider such change of coordinate system to compare relative z-values of RC, RA, and RD with C, A, and D. On the other hand, x-values for light spots A and B vary if the coordinate system moves or rotates though they always have the same z-value so that when the camera system moves or turns to take the image of A and B directly, their relative positions in the x-value vary as shown by Figure 4 below.

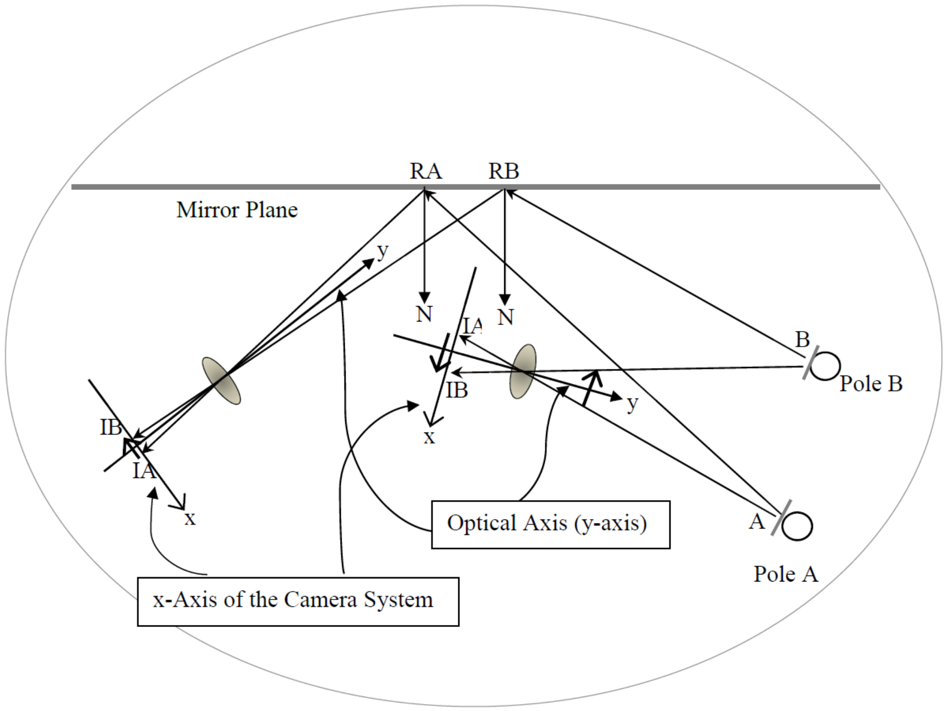

Figure 4 Different coordinate systems to take the mirror image and direct images.

Moreover, light points A and B are both on one side of each pole so that for the y-axis (optical axis) to pass through between A and B, the camera system may have to move and turn to face the same sides of those poles as when it meets the reflection points (RA and RB) so that the coordinate system may have to move and rotate in order the y-axis rotate to be directed to satisfy the needed condition. Thus, to consider positional relations in the x-axis between the mirror image and the corresponding direct image of any light source, such as the relevant case above, we may have to compare by different coordinate systems, the z-axis of which should be parallel.

Another thing to notice is that the x-values and y-values of the diagrams below of the xy-plane of the three-dimensional Cartesian coordinate system represent not only points in the xy plane, such as A and B, but also represent points in other planes than the xy-plane, each of those is the plane of incidence. That is because we consider only x-values in the three-dimensional Cartesian coordinate system. Figures other than Fig. 1 and Fig. 2 all represent the xy-plane of the three-dimensional Cartesian coordinates so that the x-value and y-value of each point in the xy-plane can indicate any point of different z-values of the same x- and y-values. Therefore, x-values in the plane of incidence relevant to C, D, E, and F, which are not parallel to the xy-plane, can be indicated. The relative angles of both sides of the Normal line should also be the same because the Normal line is always the same for any different plane of incidence. Thus, explanations relevant to light spots A and B also apply to light spots C, D, E, and F. See Figures 1 and 2 for reference for such planes.

3.4 Analyzing the line parallel to the x-axis in the xz plane

For analysis, there are conditions to confirm as follows:

1. The relative position of RA and RB relates to the relative position of IA and IB
2. The relative position of RA and RB does not relate to the relative position of A and B because the light spot that emits the ray that reflects at RA can not necessarily be A, as well, the light spot that emits the ray that reflects at RB cannot be necessarily B.

Therefore, we must determine the relative position of RA and RB (by relative x-values) without referring to the relative position of A and B (by relative x-values). We can determine the relative position of RA and RB in relation to the relative position of PA and PB by using the principle of specular reflection. Table 2 below gives the meanings of letter symbols used in the following analysis and Figures.

Table 2 Definitions of letter symbols

Letter Symbol	**Meaning**
A	Light Spot A
B	Light Spot B
RA	Reflection Point for A
RB	Reflection Point for B
RC	Intersecting point for the mirror plane and the y-axis (The optical axis).
PA	Passing Through Point for the Ray from A in a line that is parallel to the Mirror Plane and includes PB,
PB	Passing Through Point for the Ray from B in a line that is parallel to the Mirror Plane and includes PA
PC	Passing Through Points for the Ray that reflects at RC and goes through LC
LC	Optical Center of the Lens
N	Normal Line for the Mirror Plane
IA	Real Image of Light Spot A Formed on the Image Plane (xz-Plane)
IB	Real Image of Light Spot B Formed on the Image Plane (xz-Plane)
LCA	Intersecting point of the line parallel to the mirror and passes LC, and the line which pass A and RA
LCB	Intersecting point of the line parallel to the mirror and passes LC, and the line which pass B and RB
MA	Midpoint of the Line Segment between LC and LCA
MB	Midpoint of the Line Segment between LC and LCB
MC	Midpoint of the Line Segment between LC and LCC
NLC	Intersecting point for the Normal line that passes LC and the mirror plane
PLC-line	Line parallel to the mirror plane and passing through LC

(1) When the mirror plane and the image plane (of the camera system) are parallel: Figure 5 depicts a situation when the image plane is parallel to the mirror plane, the x-value of B is positive, and the x-value of A is negative.

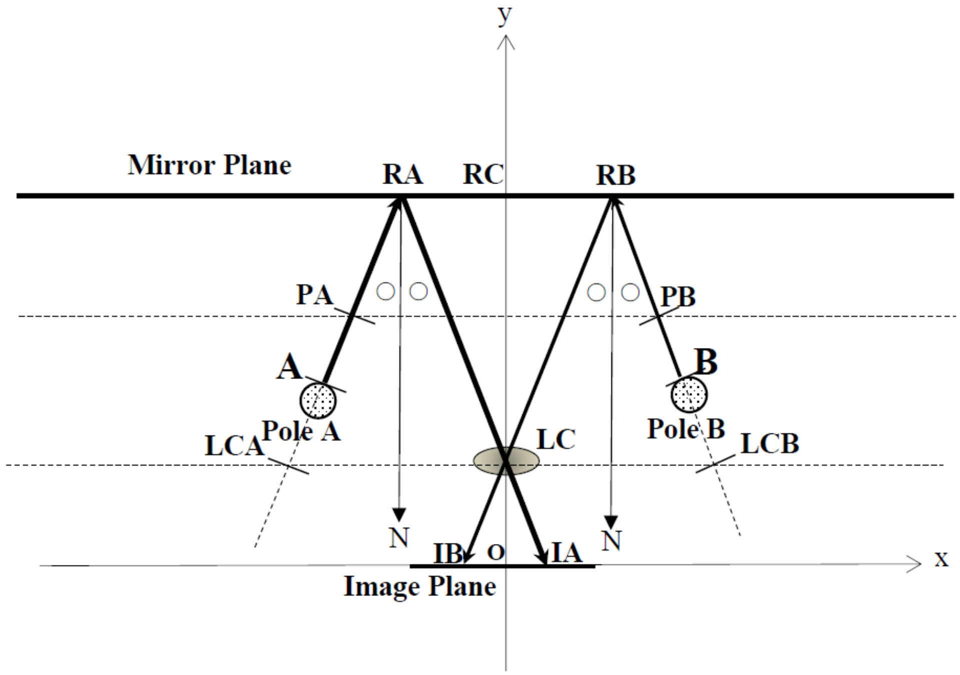

Figure 5 An optical diagram of the xy-plane when the image plane and the mirror plane are parallel.

In this diagram, when comparing the x-values of all points, the x-values of which are as follows:

x (A) < x (PA) < x (RA) < x (LC) < x (RB) < x (B)

The reason:

That x (A) < x (LC) < x (B) is from the preposition.

From the principle of specular reflection,

x (RA) = (1/2) * x (LCA) < 0. As well,

x (RB) = (1/2) * x (LCB) > 0.

Thus, x (RA) is always smaller than x (RB), so the relative position of RA and RB is always the same as that of A and B in a coordinate system when the image plane is parallel to the mirror plane.

(2) When the mirror plane and the image plane are not parallel:

Then, Fig 6 depicts a situation when the image plane of the camera system is not parallel to the mirror plane, yet the reflection points RA and RB are each other on opposite sides of the optical axis (y-axis).

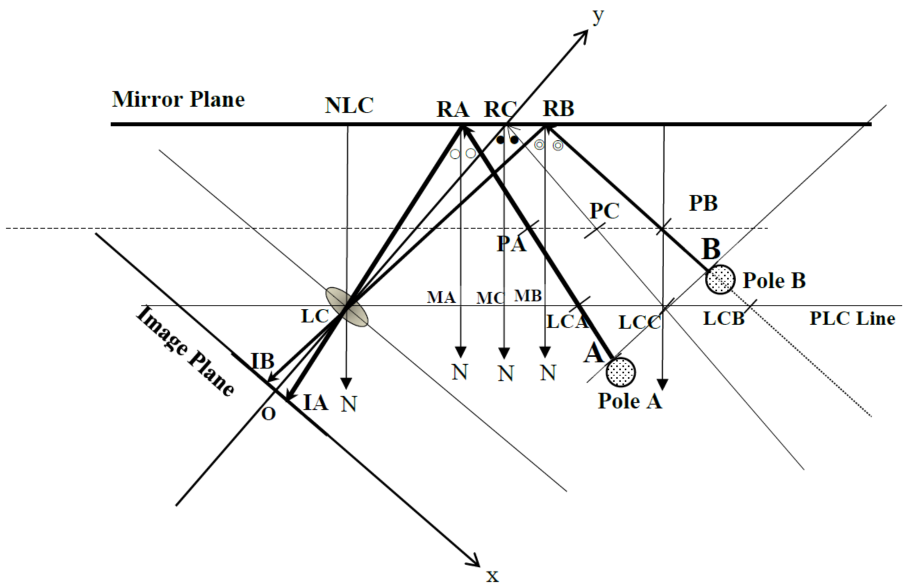

Figure 6 An optical diagram of the xy-plane when the image plane and mirror plane are not parallel.

In this condition, as in Fig 6, the relative x-value of A and B can be either plus or minus so that the relative value of PA and PB can be used to compare their relative reflection points (RA and RB) with them. Then, A, PA, RA, and LCA are all in a line, and B, PB, RB, and LCB are all in a line. When LC – LCA, LC – LCC, and LC – LCB represent distances for each two points connected by the hyphen, relative distances are as follows:

When x (PB) > x (PA)

LCA – LC < LCC – LC < LCB – LC, from the definition. And,

MA – LC < MC – LC < MB – LC because their values are 1/2 of LCA – LC, LCC – LC, and LCB – LC, respectively.

On the other hand, from the principle of specular reflection, the Normal line at RA, RC, and RB passes MA, MC, and MB, respectively. Therefore, distances between NLC and RA, NLC and RC, and NLC and RB are equal to MA – LC, MC – LC, and MB – LC, respectively. The x-value of RC is zero from the preposition so that:

x (RA) < 0, and x (RB) > 0, so that x (RB) > x (RA).

Thus, the relative position of RA and RB in the x-axis is the same as that of PA and PB, so x-values for IA and IB reverse with those for RA and RB according to the image formation principle of the convex lens.

(3) From the above results, we can deduce the following: The consideration above proves that the relative positions in the x-values of reflection points are the same as those of the original light spots in the same coordinate system. However, the camera system can have to move and rotate to take the image of those light spots directly, without mirror reflection. In the condition depicted in Figure 5, in order to take the image directly from the same front as when reflected by the mirror, the direction of the y-axis must reverse (rotate 180˚) so that the relative x-values of PA and PB, and those of A and B must reverse. In the condition depicted in Figure 6, the coordinate system must translate and rotate in order for the y-axis to be directed so that the lens system faces A and B from the same angle as when facing RA and RB, resulting in the relative x-values of A and B becoming reversed to the relative x-value for RA and RB. In order for the lens system to face A and B, the y-axis may not necessarily be directed between A and B by

the same angle as when directed between RA and RB, but in any case, between the ray reflected on the mirror surface to reach the image plane through the optical center of the lens and the ray directly from the same light spot, arise some angle in xy-plane because the relative position of images of two light spots (A and B) in the x-axis reverses between by the reflected ray and direct ray from the same light spots. This principle is also relevant to the condition in which the mirror image and direct image of the same object can be taken in the image plane of the camera system at a time. It is self-evident that the relative position of the images of objects taken directly in a camera system cannot differ between when the mirror image of them cannot taken and when the mirror image of the same objects is also taken on the same image plane, though in this condition, the coordinate system does not rotate nor translate, as illustrated in Figure 7.

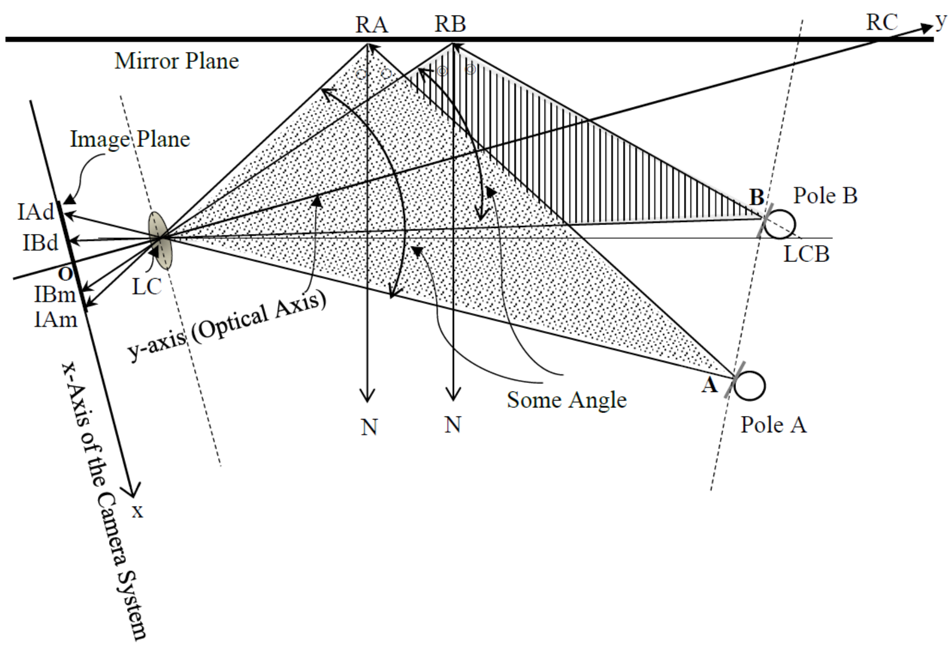

Figure 7 Optical diagram on the xy-plane when both the mirror and direct images are taken.

4. General Results

4.1 Summary of the analyses

The result of the above analysis can be summarized as follows:

Suppose a camera system with a convex lens and its image plane can move so as to rotate around one axis and translate only in a plane perpendicular to the axis, and the axis itself cannot rotate or reverse. When the y-axis of the Cartesian coordinate system is applied to the optical axis of the lens, the x-axis and the z-axis can be applied to the image plane. When only the z-axis of the camera is always perpendicular to the Normal line of a mirror plane, and the object to be taken by the camera is within the scope of the camera's rotation around the z-axis and translation, the taken image of that object on the image plane by rays reflected by the mirror plane, and another image of the same object taken by the same camera by rays that directly reach the image plane without mirror reflection are different each other in that the relative position of each two-point of the image reverses in the x-axis but does not reverse in the z-axis, whether or not the two images are taken in the image plane at a time. However, that is when the image plane is not parallel to the mirror, so if both the x- and z-axes are perpendicular to the Normal line, those axes cannot be identified. Therefore, we must consider other factors than the relative positions of the mirror and image plane.

4.2 As for Positions of the mirror and image plane

The position of the image plane against the mirror plane should be either parallel or not parallel to the mirror plane. Any plane not parallel to the mirror plane crosses with the mirror plane at one line so that such a line in the image plane can be the z-axis. On the other hand, when the image plane is parallel to the mirror plane (when the camera system directly faces the mirror), both the x-axis and z-axis are perpendicular to the Normal line so that x- and z-axes cannot be identified without using other factors. In such a condition, the xy-plane (y-axis is equal to the optical axis) should include the rays starting from the main part of the object to reach the image plane without mirror reflection through the lens because, in such a condition, the optical axis of the lens (y-axis) must rotate around the z-axis (in xy-plane) in order to take the object's image. However, such a condition is limited, and in order to take the image of the camera itself, another camera should be needed. Therefore, the x-axis cannot be identified in the physical process when the mirror and image planes are parallel.

4.3 Human factor as the essential element

The mirror reversal phenomenon itself is a cognitive phenomenon on the visual image. The visual image is not a physical object but cannot arise without the physical mechanism of the eye, which has the same mechanism as the camera system.

It is clear that the above camera system can represent the human eye. The y-axis should be the optical axis of the crystalline lens, the z-axis typically represents the vertical line of the head, and the x-axis represents the right-left line, though x- and z-axes are exchangeable according to the condition. However, we do not view the real image formed on the retina as such but perceive the object as the visual image through the real image formed in the eye. In other words, the perceived image is within the anisotropic space, whereas the real image formed in the eye is in the isotropic space. Nevertheless, relative differences between two real images formed on the retina can be perceived in the anisotropic space. Thus, the function of the specular reflection of the mirror as the cause of the mirror reversal has been verified.

As for specific conditions, for example, when the optical axis of the camera system (y-axis) must move and rotate to take the direct image of the object, the camera system cannot move automatically. Especially when the image plane (xz-plane) is parallel to the mirror plane, and the object is the camera system itself, another camera system that has an angle of 180˚ to the original camera must be needed, but any axis on the image plane can be both the z-axis and x-axis because either axis is perpendicular to the Normal line. In either case, what moves the camera and sees the image is the human, and generally, the camera's axes coincide with the human eye, head, and, or body as a whole. Therefore, in most instances, when the mirror plane is upright, the z-axis coincides with the top-bottom axis, and the x-axis coincides with the right-left axis of the human. On the other hand, when the standing person views the horizontal mirror plane, for example, the water surface, the right-left axis of the human should be the z-axis, which is perpendicular to the Normal line.

For detailed conditions relevant to the cognitive aspect, please refer to my last works: Tanaka, J. (2021, 2022).

4.4 As for the term M-D-Rotation

As mentioned in the Introduction, The M-D-Rotation (Angle, Axis, or Plane) mechanism is the mechanism to determine the axis in which the observer recognizes the reversal. Now that the physical mechanism of the mirror reversal has been deduced directly from the specular reflection by the mirror, the M-D-rotation mechanism might seem no use. Moreover, the word "M-D-Rotation" gives the impression of a mechanical movement. Indeed, the optical axis of the lens must translate and rotate around the z-axis sometimes to take the direct (not by mirror-reflected rays) image of the object. However, the camera can often take both images made by mirror-reflected and direct rays at a time. In both cases, some angle at the optical center of the lens and in the plane of incidence arises between the ray that reaches the image plane after the mirror reflection and the ray that reaches the image plane directly, without reflection. Figure 7 indicates this situation in the xy-plane.

It is important to know that the angle above is not the reflection angle nor the complementary angle of it though it arises in the plane of incidence. The angle also differs from the rotation angle of the optical axis when the rotation angle of the optical axis (y-axis) arises. The angle is made by three points that are the light spot, its reflection point on the mirror plane, and the optical center of the lens between two rays. We may define the angle made by the direct ray and reflected ray at the optical center of the lens. The author names this angle as "D-R-Rays angle (Direct and Reflected rays angle)" for the temporary use.

Any light spot is out of the mirror plane, and any reflection point is on the mirror plane, and the optical center of the lens is at a fixed point with different x- and y-values from the other two points in the coordinate system so that all of them have different x- and z-values each other. Therefore, every such triangle in a plane of incidence corresponds to the triangle on the xy-plane that has the same x- and y-values as the former triangle, and, any rotation angle around the optical center of the lens on the xy-plane correspond any rotation angle around z-axis. However, this rotation angle around the z-axis differs from the M-D-Rotation angle in that this rotation angle around the z-axis varies for each light spot on the object and its image, whereas the M-D-Rotation angle is only one for the image plane at a time. In a sense, the D-R-Rays angle is the cause, whereas the M-D-Rotation angle is the result. Moreover, rotating the optical axis of the camera system is a human-induced operation. Thus, the term M-D-Rotation might be inappropriate; however, even when both the mirror image and direct image can be taken in the image plane at a time, the rotation of the optical axis (z-axis) should be needed in order to take the direct image of the same face of the object as the mirror image. Thus, the term M-D-Rotation would be conveniently adopted.

5. Discussion

In this study, there were two provisions to analyze the problem:
One is that the word "image" in the optical system does not mean the "virtual image" but the "real image" as the term of the optics.

The other is that we should consider two different images of one object: the image made by the mirror-reflected rays and the image made by rays from the object without the mirror-reflection.

Those two provisions have been the essential conditions to accomplish this work. All previous works before the author's previous work have been studied based on the virtual image when analyzed optically and geometrically.

The mirror reversal phenomenon is not a phenomenon of the physical light or object but of the visual image. The visual image cannot be analyzed physically, so we must find any aspect that connects the visual image and any physical existence. It must be the real image formed on the image plane of the eye because the virtual image is optically no more than the geometric supposition. Many investigators tried to analyze the virtual image geometrically on the virtual image depicted on the paper. The virtual image, in fact, should be drawn by deducing from the image on the image plane. However, almost in all papers, the virtual image of the mirror image has been drawn without the real image and is generally depicted from any viewpoint and conditions, such as positions, directions, and eyesight other than the observer's, so that it cannot represent the observer's specific viewpoint.

This study succeeded in analyzing physical mechanisms using the Cartesian coordinate system, which should be because I noticed the eye as the central optical mechanism for the phenomenon. As the eye can be replaced by the camera system so long as the optics are concerned, we can exclude directional notions such as top-bottom, front-back, and right-left to make it possible to utilize the Cartesian coordinate system. That would be because the Cartesian coordinate system is adaptable only to the isotropic space applicable to analyze the physical process.

6. Conclusion

Answering the study's first aim, stated in 2.1, it proved that the meeting point of the physical and cognitive processes should be the real image. The real image should be a physical existence that consists of the light points. On the other hand, any image is no more than the content of the recognition. Generally, in optics, the real image should be thought of as a dual existence of both physical and cognitive.

Answering the second aim, I think the result of this study proved that the Cartesian coordinate system is suitable for the isotropic space set down by E. Cassirer. It should be noticed that the concept of isotropic and anisotropic spaces is rather philosophical or epistemological. Thus, I would like to conclude this paper here.

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Disclosures

The author declares no conflicts of interest.

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