# A MATHEMATICAL MODEL FOR THE PATTERN OF COVID-19 POST-VACCINATION MORTALITY AND EXCESS MORTALITY

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ABSTRACT. Using the Erlang distribution, this article gives a mathematical model for the pattern of post-vaccination mortality and discusses excess mortality based on this interpretation.

## INTRODUCTION

Reference [2] presents a histogram with the number of days from COVID-19 vaccination to death on the horizontal axis and the number of deaths on the vertical axis, based on a report from the Ministry of Health, Labour and Welfare (MHLW). Furthermore, we can find a similar histogram whose peak appears on the second day and then decreases in [6, Figure S1]. In the natural sciences, it is important to give mathematical interpretations to natural phenomena. For example, projectile motion and wave motion are mathematically described by parabolas and trigonometric functions, respectively. However, there are no mathematical explanation of the histogram until now. Here we give a mathematical interpretation or model for the histogram and discusses excess mortality based on this model. More precisely, this article suggests that the histogram described in Figure 2, reference [2], can be approximated by the modified Erlang distribution. Note that the Erlang distribution of the parameters $(n, \lambda)$ is a model in which small changes occur $n$ times with a constant average incidence $\lambda$ to cause a failure. When dealing with short-term deaths within 5 days, Stirling's formula and numerical computations indicate that the parameter $n$ in the Erlang distribution is smaller than 5, which suggests sudden death of vaccine recipients. Moreover, we discuss very long-term deaths, which may be related to excess mortality by changing the parameters of the Erlang distribution.

This paper is structured as follows: Section 1 introduces the probability distribution based on references [8] and [7] related to this subject; Section 2 gives a review of Kiyoshi Itô's argument in [4, Chapter 3.4] and provides an analogue or generalization of his argument; Section 3 explains mathematically that the histogram shown in Figure 2, reference [2], can be approximated by a modified Erlang distribution; Section 4 contains some notes on how to choose parameters; and Section 5 discusses possible explanations for excess mortality if the modified Erlang distribution model is correct.

## 1. ASSOCIATED DISTRIBUTION

An exponential distribution is a distribution in probability theory and statistics that describes, for example, the time intervals between events according to a Poisson process (a process in which events occur continuously, independently, and with a constant average incidence). Because of this property, the exponential distribution is often used as a model of random failure. The mean, mode, variance, probability density function, and cumulative distribution function of the exponential distribution with the parameter $\lambda$ are as follows, respectively:

$$\frac{1}{\lambda}, \qquad 0, \qquad \frac{1}{\lambda^2}, \qquad \lambda e^{-\lambda x}, \qquad 1 - e^{-\lambda x}.$$

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It is known that for $n$ random variables $X_1, \dots, X_n$ which are independent from each other and follow the exponential distribution of parameter $\lambda$, the sum of these random variables $S_n = X_1 + \cdots + X_n$ follows the Erlang distribution with the parameters $(n, \lambda)$. Therefore, the Erlang distribution of the parameters $(n, \lambda)$ is a model in which small changes occur $n$ times with a constant average incidence $\lambda$ to cause a failure. Clearly, the Erlang distribution of $(1, \lambda)$ is an exponential distribution with a parameter $\lambda$. The mean, mode, variance, probability density function, and cumulative distribution function of the Erlang distribution with the parameters $(n, \lambda)$ are as follows:

$$\frac{n}{\lambda}, \qquad \frac{n-1}{\lambda}, \qquad \frac{n}{\lambda^2}, \qquad \frac{\lambda^n}{(n-1)!} x^{n-1} e^{-\lambda x}, \qquad 1 - e^{-\lambda x} \sum_{l=0}^{n-1} \frac{(\lambda x)^l}{l!}.$$

Since the Erlang distribution considers the failure with respect to a continuous time course, we consider discrete cases such as the first day, the second day, etc. From the cumulative distribution function of the Erlang distribution, the probability of small changes $n$ times by day $k$, where $n$ and $k$ are natural numbers, is given by

$$1 - G(m, \lambda, k), \qquad G(m, \lambda, k) := e^{-\lambda k} \sum_{l=0}^{n-1} \frac{(\lambda k)^l}{l!}.$$

Since the probability of small changes $n$ times on day $k$ is obtained by subtracting the probability of small changes $n$ times by day $k-1$ from the probability of small changes $n$ times by day $k$, the probability of small changes $n$ times on day $k$ can be expressed as:

$$G(m, \lambda, k-1) - G(m, \lambda, k).$$

For $k \le x < k+1$, let

$$\eta(m, \lambda, x) := G(m, \lambda, k-1) - G(m, \lambda, k), \qquad \theta(m, \lambda, x) := 1 - G(m, \lambda, k).$$

Then, a distribution in which the probability density function is $\eta(m, \lambda, x)$, that is, whose cumulative distribution function is $\theta(m, \lambda, x)$, and both the probability density and the cumulative distribution functions are 0 when $x < 1$, is called a modified Erlang distribution.

## 2. Itô's argument

In this section, we summarize and generalize Itô's argument appeared in in [4, Chapter 3.4]. Let $N(t)$ the number of radioactive atoms at time $t$. Then, the radioactive decay is described by the following differential equation

$$\frac{d}{dt} N(t) = -\alpha N(t), \qquad N(0) = N,$$

where $\alpha$ is a disintegration constant (or exponential decay constant). As Kiyoshi Itô pointed out, the number $N(t)$ is always an integer, namely, $N(t)$ is a discontinuous function. However, it is well-know that any discontinuous function is non-differentiable. Hence, this argument is not mathematically rigorous.

To avoid this non-differentiability, Itô discussed as follows. Let $p(t)$ the probability that each atom survives (does not decay) until time $t$. Then we have

$$\frac{d}{dt} p(t) = -\alpha p(t), \qquad p(0) = 1.$$

By solving the differential equation, we have

$$p(t) = e^{-\alpha t}.$$

Assign number to each atom and let $X_n(t) = 1$ or 0 based on whether the atom $n$ survives or not by time $t$. Then the total number of surviving atoms $N(t)$ by time $t$ is the stochastic process expressed as

$$N(t) := \sum_{n=1}^{N} X_n(t).$$

By taking the means of both sides of the equation, we obtain

$$\mathrm{E}\big(N(t)\big) = \sum_{n=1}^{N} \mathrm{E}\big(X_n(t)\big) = \sum_{n=1}^{N} p(t) = Np(t) = Ne^{-\alpha t}.$$

Let $N$ be sufficiently large. Then, the numbers $N(t)$ and $\mathrm{E}(N(t))$ are very close to each other according to the law of large numbers [9] if we ignore the extremely small probability. Kiyoshi Itô (known as the founder of so-called Itô calculus) said that

> In the former argument, we forcibly applied theory of ordinary differential equations. However, we can consider naturally the latter argument in theory of probability. Moreover, the later argument is much closer to the truth in the position on the scientific understanding of radioactive decay.

Next, we consider an analogue or generalization of his argument. Recall that $p(t)$ is the probability that each atom does not decay until time $t$. We define $q(t)$ by the probability that each atom decays until time $t$. Then, clearly we have

$$q(t) = 1 - p(t) = 1 - e^{-\alpha t}.$$

It should be noted that the right hand side of the formula above coincides with the cumulative distribution function of the exponential distribution with the parameter $\alpha$. Let $Y_m(t) = 1$ or 0 based on whether the atom $m$ decays or not by time $t$. Then the total number of decayed atoms $M(t)$ by time $t$ is the stochastic process given by

$$M(t) := \sum_{m=1}^{M} Y_m(t).$$

Taking the means of both sides of the equation above, we obtain

$$\mathrm{E}\big(M(t)\big) = \sum_{m=1}^{M} \mathrm{E}\big(M_l(t)\big) = \sum_{m=1}^{M} q(t) = Mq(t) = M\big(1 - e^{-\alpha t}\big).$$

Therefore, the exponential distribution is a mathematical model of the radioactive decay.

Now we consider the case 500 atoms has an exponential decay constant $\alpha$ and 200 atoms has an exponential decay constant $\beta$. Let $Y_m(t) = 1$ or 0 based on whether the atom $m$ with the disintegration constant $\alpha$ decays or not by time $t$ and let $Z_n(t) = 1$ or 0 based on whether the atom $n$ with the disintegration constant $\beta$ decays or not by time $t$. Then the radioactive decay of these 700 atoms are described by the following stochastic process given as

$$\sum_{m=1}^{500} Y_m(t) + \sum_{n=1}^{200} Z_n(t).$$

By taking the means of the stochastic process above, we have

$$\sum_{m=1}^{500} \mathrm{E}\big(Y_m(t)\big) + \sum_{n=1}^{200} \mathrm{E}\big(Z_n(t)\big) = 500\big(1 - e^{-\alpha t}\big) + 200\big(1 - e^{-\beta t}\big).$$

Therefore, we can handle the case when two types of atoms are mixed by modifying Itô's argument. Obviously, there is no need to limit the types of mixed atoms to two types, namely, there is no problem with any number of types.

## 3. MATHEMATICAL MODEL FOR PATTERNS OF POST-VACCINATION MORTALITY

Recall that the Erlang distribution of the parameters $(n, \lambda)$ is a model in which small changes occur $n$ times with a constant average incidence $\lambda$ to cause a failure and the Erlang distribution of $(1, \lambda)$ is an exponential distribution with a parameter $\lambda$. Consider a model in which a small change occurs in the body after vaccination, and death occurs after $n$ instances of the change. Then the probability $p(t)$ that each vaccine recipient will die by time $t$ can be considered to follow the Erlang distribution as discussed in Section 1, and is derived as follows:

$$p(t) = 1 - e^{-\lambda t} \sum_{l=0}^{n-1} \frac{(\lambda t)^l}{l!}.$$

Note that the right hand side of the formula above is the cumulative distribution function of the Erlang distribution with the parameters $(n, \lambda)$. Clearly, the above equation describes the situation in continuous time, while Figure 2 in [2] shows the discrete time in days on the horizontal axis. Therefore, it is necessary to use a modified Erlang distribution. In summary:

> The percentage of people who die after vaccination causes a change at the rate $\lambda$ and
> the number of changes reaches $n$ on day $k$ can be approximated by
> the modified Erlang distribution with the parameters $(n, \lambda)$.

This leads us to believe that the histogram Figure 2 described in reference [2] is approximated by the modified Erlang distribution. This can be validated by the following numerical computation. First, we divide Figure 2 into short-term mortality (within 5 days), medium-term mortality (6 to 15 days), and long-term mortality (16 to 30 days), and assume that each of these follows the modified Erlang distribution with parameters $(3, 1.6)$, $(3, 0.44)$, and $(3, 0.14)$. Then their probability density functions are as follows. The blue circles, orange squares, and green rhombus correspond to the modified Erlang distribution (MED) with the parameters $(3, 1.6)$, $(3, 0.44)$, and $(3, 0.14)$, respectively (FIGURE 1).

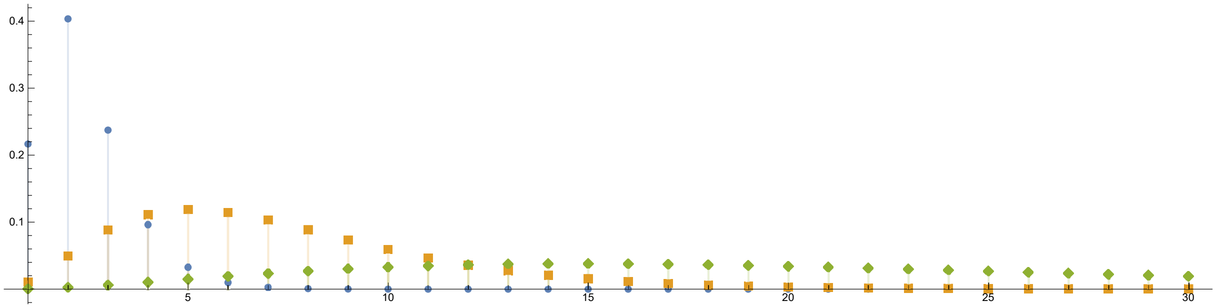

FIGURE 1. MED with parameters $(3, 1.6)$, $(3, 0.44)$, and $(3, 0.14)$, $1 \leq k \leq 30$

As this graph shows, death within 5 days is also associated with intermediate or long-term mortality. For the reasons given above, although the numbers of short-term, medium-term, and long-term deaths in Figure 2, reference [2] are 457, 220, and 66, respectively, we assume that 303 people follow MED$(3, 1.6)$, 320 people follow MED$(3, 0.44)$, and 90 people follow MED$(3, 0.14)$, where MED$(n, \lambda)$ is the modified Erlang distribution with the parameter $(n, \lambda)$. Namely, we consider the stochastic process

$$\sum_{l=1}^{303} X_l(t) + \sum_{m=1}^{320} Y_m(t) + \sum_{n=1}^{90} Z_n(t), \tag{$\flat$}$$

where $X_l(t) = 1$ or $0$ based on whether the atom $l$, which follows MED$(3, 1.6)$, decays or not, $Y_m(t) = 1$ or $0$ based on whether the atom $m$ following MED$(3, 0.44)$ decays or not, and let

$Z_n(t) = 1$ or 0 based on whether the atom $n$ following MED$(3, 0.14)$ decays or not by time $t$. By taking the means of the stochastic process above, we have

$$\begin{aligned}&\sum_{l=1}^{303} \mathrm{E}\big(X_l(t)\big) + \sum_{m=1}^{320} \mathrm{E}\big(Y_m(t)\big) + \sum_{n=1}^{90} \mathrm{E}\big(Z_n(t)\big)\\ &= 303\theta(3, 1.6, t) + 320\theta(3, 0.44, t) + 90\theta(3, 0.14, t),\end{aligned} \tag{$\sharp$}$$

where $\theta(n, \lambda, x)$ is the cumulative distribution function of MED$(n, \lambda)$ defined in Section 1. It should be emphasized that $(\flat)$ and $(\sharp)$ are close to each other by the law of large numbers if we ignore the extremely small probability (see Section 2). Furthermore, since the attributes of vaccinated people differ depending on their age and the presence or absence of underlying diseases, it is natural that the duration of death differs. In other words, when the same medicine is administered to uniform experimental animals, it is not necessary to choose three disintegration constants as above, and it is sufficient to specify one disintegration constant. The reason for setting $n = 3$ will be explained in the next section. The orange line below represents the line from Figure 2 in reference [2], and the blue line represents the line from the right hand side of $(\sharp)$ (Figure 2).

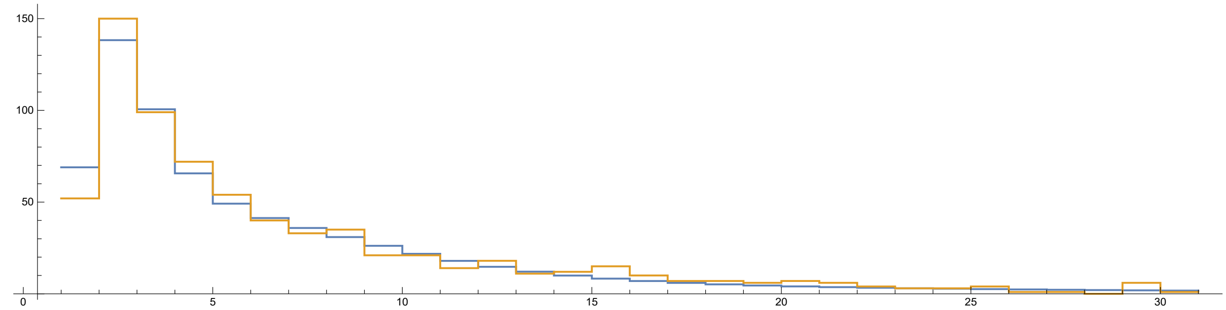

Figure 2. Comparison of $(\sharp)$ with [2, Figure 2], $1 \leq k \leq 30$

In Figure 2 in reference [2] (see Reference data 1), it can be seen that similar graphs are obtained when only the data reported on May 26, June 9, June 23, July 7 and July 21 are used, in which the peak appears on the second or third day and then decreases. It is reasonable to suspect that $\lambda = 1.6$, the rate at which vaccine-induced changes occur, is too high. However, this is because the histogram Figure 2 described in reference [2] was originally provided based on the ”Summary of Events Reported as Death after COVID-19 Vaccination” published by MHLW, and it is inevitable that the population had a high rate of small changes caused by the vaccination. In Figure 2 in reference [2], there is a possibility that some recipients were recorded as ”dead on the second day after vaccination if death was confirmed the morning after the day of vaccination”. Furthermore, if the vaccine was administered in the evening and the recipient died the next morning, less than 24 hours had passed since vaccination. Hence, there is a large difference between deaths recorded on the first day and the second day.

We can find a similar histogram whose peak appears on the second day and then decreases in Figure S1 in reference [6] (see Reference data 2). In addition, similar graphs are obtained when only the data caused by BNT162b2 Vaccine and mRNA-1273 vaccine. Furthermore, we can find a similar shape in [1, Figure 1] with the number of days from COVID-19 vaccination to symptom on the horizontal axis and the number of patients of with myocarorditis in Korea on the vertical axis. Therefore, this phenomenon is not unique to post-vaccination mortality in Japan, but is expected to occur in many side effects from COVID vaccine all over the world.

## 4. REASON FOR SETTING $n = 3$ AND ITS MEANING

This section explains that when $n$ is large, short-term mortality within 5 days is quite different in shape from Figure 2 in reference [2]. To do this, we first consider the mode $\rho$ of the Erlang distribution. Based on Figure 2 in reference [2], the $\rho$ should satisfy:

$$1 < \rho := \frac{n-1}{\lambda} < 3.$$

If $n \geq 2$ the variance of the Erlang distribution is

$$\frac{n}{\lambda^2} = n\frac{\rho^2}{(n-1)^2} < \frac{9n}{(n-1)^2}.$$

This implies that when $n$ is large enough, the variance of the Erlang distribution is very small. The modified Erlang distribution is thought to be similar. However, Figure 2 in reference [2] cannot be approximated unless the distribution has a relatively large variance. This argument is more effective when $\rho$ is close to 1 because the variance of the Erlang distribution is small. Conversely, if $\rho$ is close to 3, we discuss as follows: when $\rho$ satisfies the above condition, the number of deaths up to day $\rho/3$ is bounded by

$$\int_0^{\rho/3} \frac{\lambda^n}{(n-1)!} x^{n-1} e^{-\lambda x} dx \leq \int_0^{\rho/3} \frac{\lambda^n x^{n-1}}{(n-1)!} dx = \frac{1}{n!}\frac{(\lambda\rho)^n}{3^n}$$

from $e^{-x} \leq 1$ for $x \geq 0$ and the probability density function of the Erlang distribution. Let $F(n)$ be the right-hand side of the above equation, namely, we put

$$F(n) := \frac{1}{n!}\frac{(\lambda\rho)^n}{3^n} = \frac{1}{n!}\frac{(n-1)^n}{3^n}.$$

Then we have the following:

$$F(19) = 0.005009324157..., \qquad F(18) = 0.005669659528.., \qquad F(17) = 0.006425543377...,$$
$$F(16) = 0.007292903644..., \qquad F(15) = 0.008290908138..., \qquad F(14) = 0.009442804844...,$$
$$F(13) = 0.01077704189..., \qquad F(12) = 0.01232878279..., \qquad F(11) = 0.01414198851... .$$

Next, we consider the cases $n \geq 20$. Applying Stirling's formula, which is given by

$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \left(1 + \frac{1}{12n} + \frac{1}{288n^2} - \frac{139}{51840n^3} - \frac{571}{2488320n^4} + O\left(n^{-5}\right)\right)$$

(see [10, Speed of convergence and error estimates]), to $n!$, we have

$$F(n) = \frac{(n-1)^n}{n3^n}\frac{1}{\sqrt{2\pi(n-1)}}\left(\frac{e}{n-1}\right)^{n-1}\left(1 + O(n^{-1})\right) = \frac{\sqrt{n-1}}{ne\sqrt{2\pi}}\left(\frac{e}{3}\right)^n\left(1 + O(n^{-1})\right).$$

Thus, when $n \geq 20$, the above value is very small by

$$\frac{1}{e\sqrt{2\pi}} = 0.1467626632..., \qquad \left(\frac{e}{3}\right)^{20} = 0.1391440191... .$$

Therefore, it is not necessary to consider when $n \geq 11$ because the number of deaths by day $\rho/3$ is too small if $1 < \rho < 3$. By gradually decreasing $n$ from $n = 10$, it can be understood by numerical computation that $n = 3$ is optimal. In fact, for $n = 10, 9, 8, 7, 6, 5$, we obtain the following graphs (FIGURES 3, 4, 5, 6, 7, and 8). The blue circles, orange squares, green rhombus, red triangle, purple triangle, and brown circle correspond to $\rho = 1, 1.2, 1.4, 1.6, 1.8, 2$, respectively.

Considering the balance of the height differences on the first, second, third, fourth, and fifth days, it seems that Figure 2 in reference [2] cannot be approximated by these figures. Therefore, $n = 2, 3, 4$ are candidates, but here, $n = 3$ is taken as the median value. Actually, when $n = 2, 3, 4$, we have the following graphs (FIGURES 9, 10, and 11).

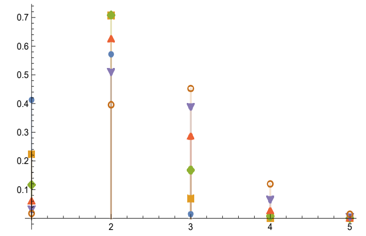

FIGURE 3. $n = 10$

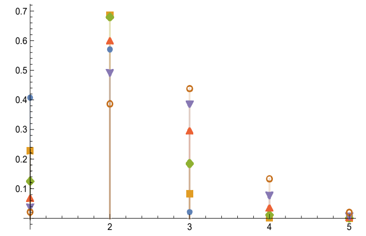

FIGURE 4. $n = 9$

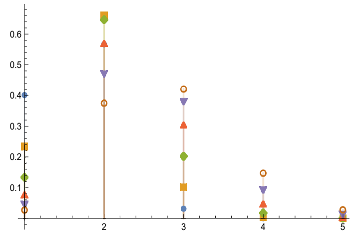

FIGURE 5. $n = 8$

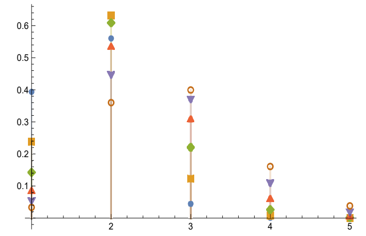

FIGURE 6. $n = 7$

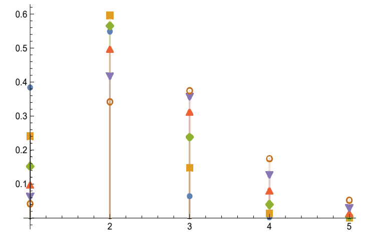

FIGURE 7. $n = 6$

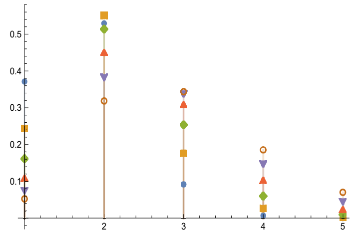

FIGURE 8. $n = 5$

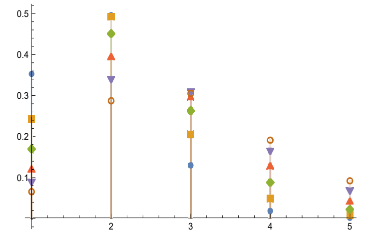

FIGURE 9. $n = 4$

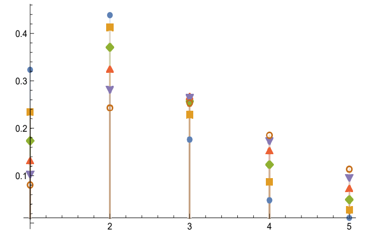

FIGURE 10. $n = 3$

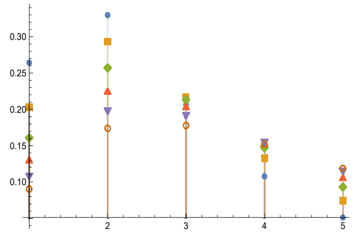

FIGURE 11. $n = 2$

In the preceding discussion, it has been assumed that death occurs after a change occurring $n$ times, but the above discussion has shown that the optimal value of $n$ is 3. That is, when dealing with short-term deaths within 5 days, it would be expected that $n$ may be greater than 3, but not greater than 10. This has significant implications: death from three changes means a dramatic worsening of the condition, which suggests sudden death. In fact, the 28-year-old man described in reference [3] died suddenly of myocardial rhabdomyolysis 5 days after the second vaccination. Furthermore, it is reported in [5] that a 14-year-old girl died suddenly of fatal multi-organ inflammation 2 days after the third dose of the BNT1262b2 mRNA COVID-19 vaccine. Therefore, this mathematical model is highly rational.

## 5. EXCESS MORTALITY

It should be noted that, unlike the previous discussion, this section is not based on specific data from reference [2]. The purpose of this section is to discuss what would happen to excess mortality if the modified Erlang distribution model were correct.

In the previous section, we considered deaths within a month. Now we consider deaths within a year. Therefore, let $n = 3$ and $\lambda = 0.02$ in the modified Erlang distribution (MED). Then the mortality rate is as follows (FIGURE 12):

This graph shows that even if $\lambda = 0.02$, which is very high, the mortality peaks around 100 days after vaccination. This means that even if a person is safe for a few days after vaccination, it is not possible to rule out the possibility of death later. Also, because a long time has passed since vaccination, the causal relationship between death and vaccination may not even be discussed. As before, let $n = 3$ and $\lambda = 0.001$, and draw a graph of the cumulative distribution function (CDF) of the modified Erlang distribution with the parameters $(3, 0.001)$.

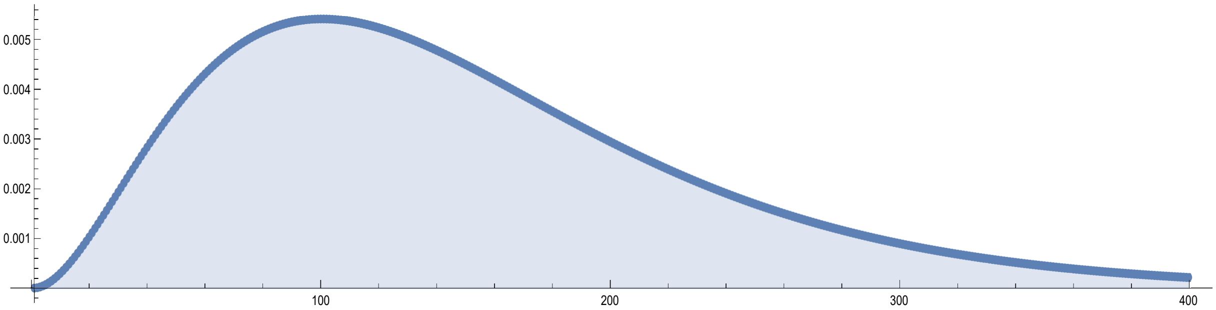

FIGURE 12. MED with parameters $(3, 0.02)$, $1 \leq k \leq 400$

This graph can then be considered to show by what date and at what rate people die in total (FIGURES 13 and 14).

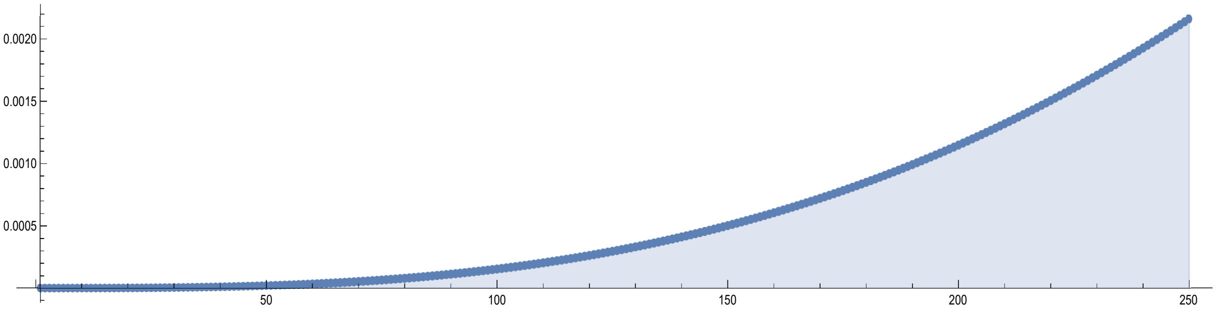

FIGURE 13. CDF of the MED with parameters $(3, 0.001)$, $1 \leq k \leq 250$

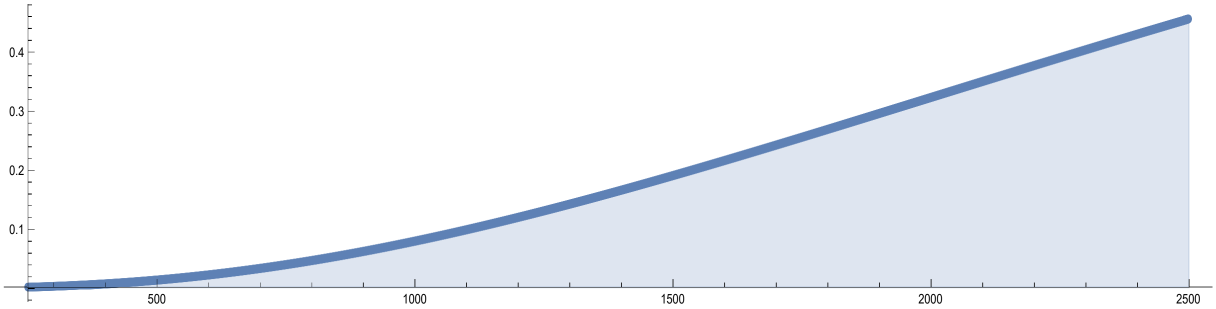

FIGURE 14. CDF of the MED with parameters $(3, 0.001)$, $250 \leq k \leq 2500$

These graphs show that, even if $\lambda = 0.001$, which is considerably high, less than 0.26% of vaccinated people will die after about 8 months (FIGURE 13), but more than 40% of vaccinated people will die after about 2500 days (FIGURE 14). If the death occurs a considerable period of time after vaccination, $n = 3$ may not hold true, so let $n = 100$ and $\lambda = 0.05$ and draw a graph of the cumulative distribution function of the modified Erlang distribution with the parameters $(100, 0.05)$. After about 50 months, the total number of deaths is almost 0, but after about 60 months, deaths gradually become apparent, and after about 2500 days, that is, about 7 years, more than 95% of vaccinated people have died (FIGURE 15). These figures show that in the model with a small n, each individual dies suddenly, but as a population the deaths occur slowly, while in the model with a large $n$, the deaths occur suddenly as a population, but each individual dies slowly. At present, the possibility that this mathematical model is associated with the drastic increase in excess mortality observed throughout the world in 2022 cannot be ruled out. Furthermore, these mathematical experiments raise the question of whether approval of the vaccines in less than 8 months from the start of a trial was correct.

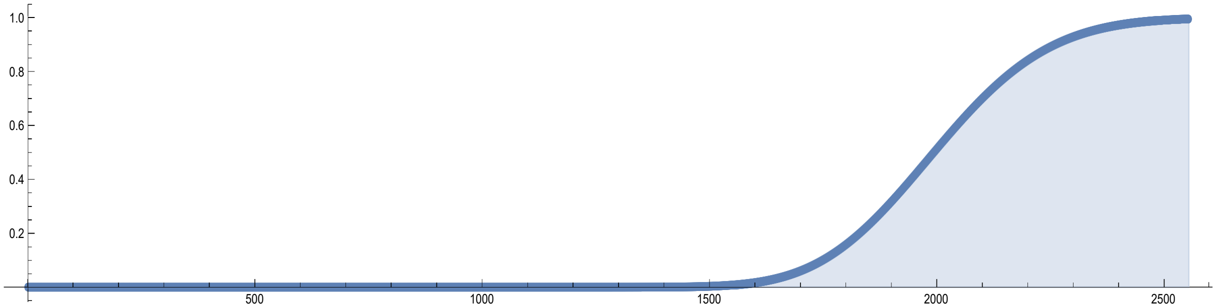

FIGURE 15. CDF of the MED with parameters $(100, 0.05)$, $1 \leq k \leq 2555$

## CONCLUSION

This article suggests that the histogram described in Figure 2, reference [2], can be approximated using the modified Erlang distribution by appropriately dividing it into short-term mortality, medium-term mortality, and long-term mortality. It is natural to think that very long-term deaths, which may be related to excess mortality, can also be explained by changing the parameters of the Erlang distribution. The proposal that patterns of post-vaccination mortality and excess mortality can be interpreted using the same mathematical principles may provide important guidance in collecting data on both.

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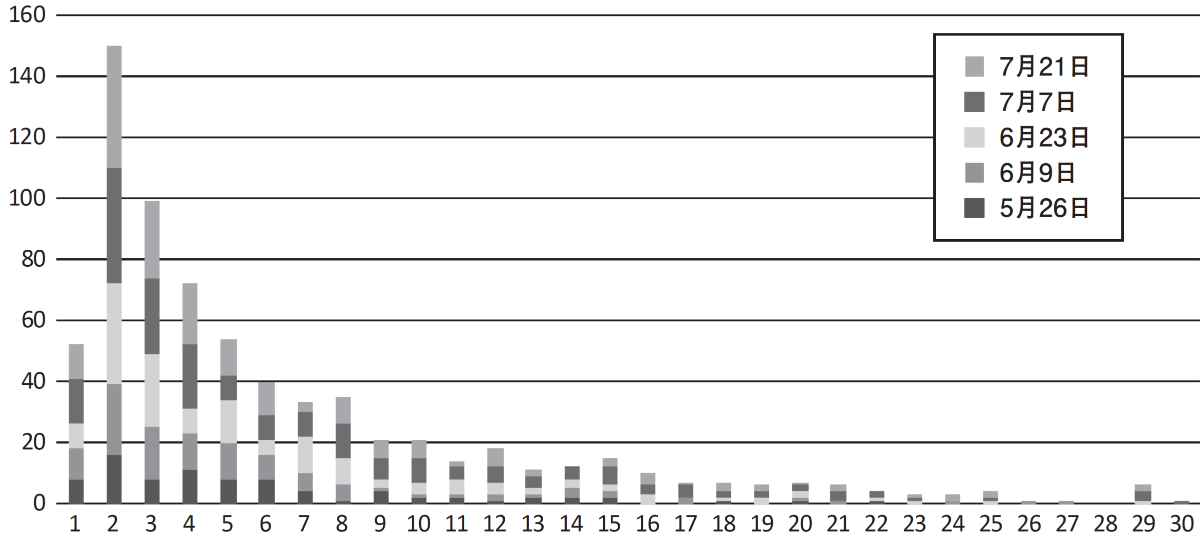
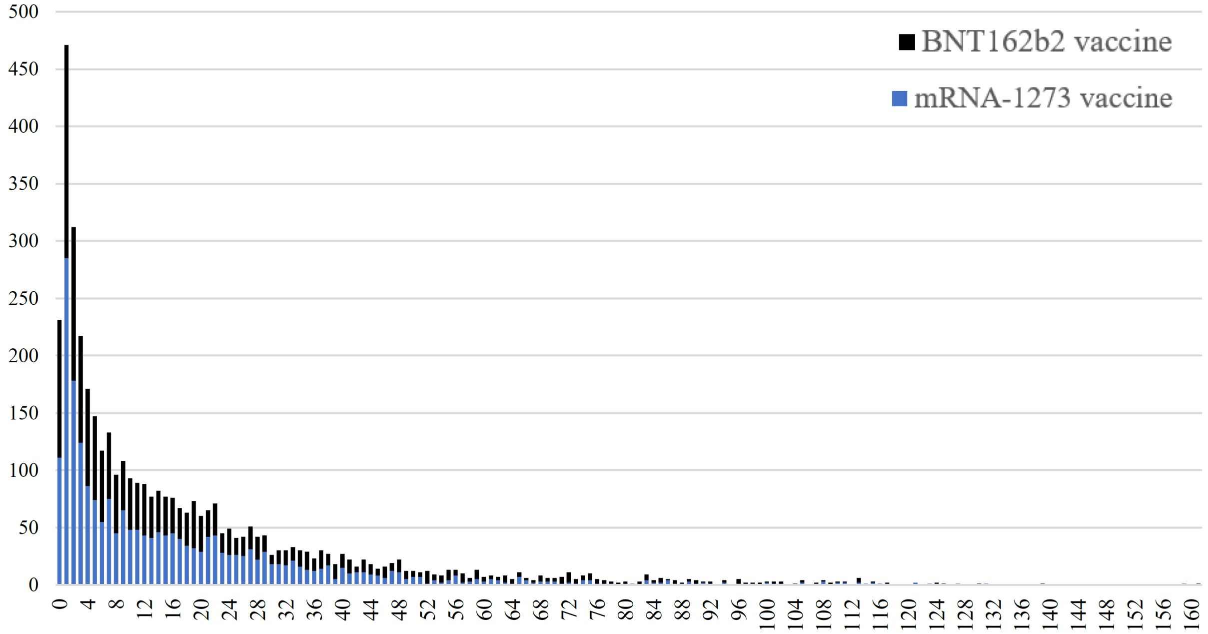
**Compliance with Ethical Standards.** The author has no conflicts of interest directly relevant to the content of this article.

**Reference data 1: Histogram described in Ref.** [2] **Figure 2.** The vertical axis is the number of deaths, and the horizontal axis is the number of days after vaccination (day 1 refers to the day of vaccination).



**Reference data 2: Histogram described in Ref.** [6] **Figure S1:** Number of reports of death per day following vaccination, by manufacturer, to Vaccine Adverse Event Reporting System (VAERS)-December 14, 2020–June 14, 2022 (day 0 refers to the day of vaccination).



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