A Mathematical Interpretation of the Pattern of COVID-19 Post-Vaccination Mortality and Excess Mortality

Takashi Nakamura

Institute of Liberal Arts and Sciences, Tokyo University of Science

nakamuratakashi@rs.tus.ac.jp

Key words: Pattern of COVID-19 post-vaccination mortality, Erlang distribution, Excess mortality

Overview

Reference [1] presents a histogram with the number of days from COVID-19 vaccination to death on the horizontal axis and the number of deaths on the vertical axis, based on a report from the Ministry of Health, Labour and Welfare (MHLW). This article gives a mathematical interpretation of the histogram and discusses excess mortality based on this interpretation.

Introduction

In the natural sciences, it is important to give mathematical interpretations to natural phenomena. For example, projectile motion and wave motion are mathematically described by parabolas and trigonometric functions, respectively. The aim of this article is to provide a mathematical interpretation of the pattern of post-vaccination mortality using the Erlang distribution.
This paper is structured as follows: Section 1 introduces the probability distribution based on references [4] and [5] related to this subject; Section 2 explains mathematically that the histogram shown in Figure 2, reference [1], can be approximated by a modified Erlang distribution; Section 3 contains some notes on how to choose parameters; and Section 4 discusses possible explanations for excess mortality if the modified Erlang distribution model is correct.

1 Associated distribution

An exponential distribution is a distribution in probability theory and statistics that describes, for example, the time intervals between events according to a Poisson process (a process in which events occur continuously, independently, and with a constant average incidence). Because of this property, the exponential distribution is often used as a model of random failure. The mean, mode, variance, probability density function, and cumulative distribution function of the exponential distribution with the parameter lambda are as follows, respectively:

$$\frac{1}{\lambda}, \qquad 0, \qquad \frac{1}{\lambda^2}, \qquad \lambda e^{-\lambda x}, \qquad 1 - e^{-\lambda x}.$$

It is known that for n random variables $X_1, \ldots, X_n$ which are independent from each other and follow the exponential distribution of parameter λ, the sum of these random variables $S_n = X_1 + \cdots + X_n$ follows the Erlang distribution with the parameters (n, λ). Therefore, the Erlang distribution of the parameters (n, λ) is a model in which small changes occur n times to cause a failure. Clearly, the Erlang distribution of $(1, \lambda)$ is an exponential distribution with a parameter λ. The mean, mode, variance, probability density function, and cumulative distribution function of the Erlang distribution with the parameters (n, λ) are as follows:

$$\frac{n}{\lambda}, \qquad \frac{n-1}{\lambda}, \qquad \frac{n}{\lambda^2}, \qquad \frac{\lambda^n}{(n-1)!}x^{n-1}e^{-\lambda x}, \qquad 1-e^{-\lambda x}\sum_{l=0}^{n-1}\frac{(\lambda x)^l}{l!}.$$

Since the Erlang distribution considers the failure with respect to a continuous time course, we consider discrete cases such as the first day, the second day, etc. From the cumulative distribution function of the Erlang distribution, the probability of small changes *n* times by day *k*, where *n* and *k* are natural numbers, is given by

$$1-G(m,\lambda,k), \qquad G(m,\lambda,k) \coloneqq e^{-\lambda k}\sum_{l=0}^{n-1}\frac{(\lambda k)^l}{l!}.$$

Since the probability of small changes *n* times on day *k* is obtained by subtracting the probability of small changes *n* times by day *k*-1 from the probability of small changes *n* times by day *k*, the probability of small changes *n* times on day *k* can be expressed as:

$$G(m,\lambda,k-1)-G(m,\lambda,k).$$

For $k \leq x < k+1$, let

$$\eta(m,\lambda,x) \coloneqq G(m,\lambda,k-1)-G(m,\lambda,k), \qquad \theta(m,\lambda,x) \coloneqq 1-G(m,\lambda,k).$$

Then, a distribution in which the probability density function is $\eta(m, \lambda, x)$, that is, whose cumulative distribution function is $\theta(m, \lambda, x)$, and both the probability density and the cumulative distribution functions are 0 when $x < 1$, is called a modified Erlang distribution.

2 Mathematical interpretation of patterns of post-vaccination mortality

Consider a model in which a small change occurs in the body after vaccination, and death occurs after *n* instances of the change. The probability $p(t)$ that each vaccine recipient will die by time *t* can be considered to follow the Erlang distribution as discussed in Section 1, and is derived as follows:

$$p(t) = 1-e^{-\lambda t}\sum_{l=0}^{n-1}\frac{(\lambda t)^l}{l!}.$$

When numbering each person 1, 2, ..., *L,* and letting $X_l(t) = 1$ or 0 based on whether a person *l* is dead or not by time *t*, the total number of recipients $L(t)$ who will die by time *t* is a stochastic process given by the following equation:

$$L(t) = \sum_{l=1}^{L} X_l(t).$$

Taking the means of both sides of the equation yields the following equation:

$$\mathrm{E}\big(L(t)\big) = \sum_{l=1}^{L}\mathrm{E}\left(X_l(t)\right) = \sum_{l=1}^{L} p(t) = Lp(t) = L\left(1-e^{-\lambda t}\sum_{l=0}^{n-1}\frac{(\lambda t)^l}{l!}\right).$$

We can find this argument in reference [3, pp. 56-58], which treats radioactive decay, if the cumulative distribution function of the Erlang distribution is replaced by that of the exponential distribution. Comparing the above discussion with the histogram Figure 2 described in reference [1], we can see that the above equation describes the situation in continuous time, while Figure 2 in [1] shows the discrete time in days on the horizontal axis. Therefore, it is necessary to use a modified Erlang distribution. In summary:

The percentage of people who die after vaccination causes a change at the rate λ and the number of changes reaches n on day k can be approximated by the modified Erlang distribution with the parameters (n, λ).

This leads us to believe that the histogram Figure 2 described in reference [1] is approximated by the modified Erlang distribution. This can be validated by the following numerical computation.

First, we divide Figure 2 into short-term mortality (within 5 days), medium-term mortality (6 to 15 days), and long-term mortality (16 to 30 days), and assume that each of these follows the modified Erlang distribution with parameters (3, 1.6), (3, 0.44), and (3, 0.14). Then their probability density functions are as follows. The blue circles, orange squares, and green rhombi correspond to the modified Erlang distribution with the parameters (3, 1.6), (3, 0.44), and (3, 0.14), respectively (Figure 1).

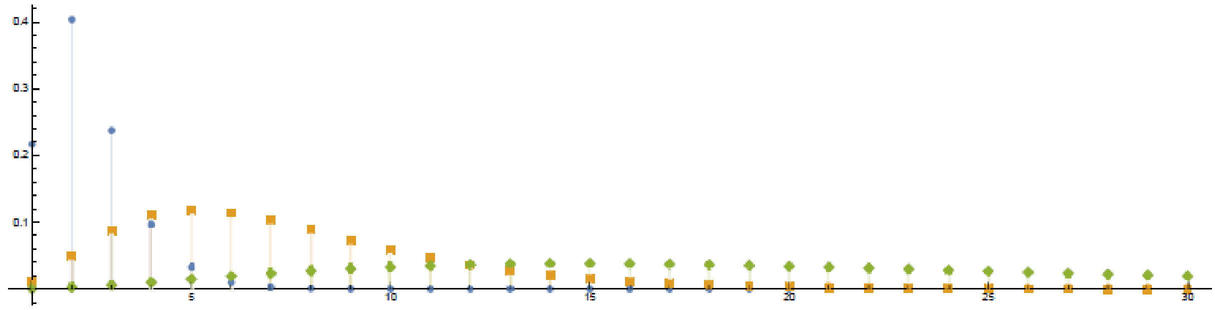


Figure 1:
Modified Erlang distribution with parameters (3, 1.6), (3, 0.44), and (3, 0.14), $1 \leq k \leq 30$

As this graph shows, death within 5 days is also associated with intermediate or long-term mortality.

For the reasons given above, although the numbers of short-term, medium-term, and long-term deaths in Figure 2, reference [1], are 457, 220, and 66, respectively, we assume that 303 people follow the modified Erlang distribution with the parameter (3, 1.6), 320 people follow the modified Erlang distribution with the parameters (3, 0.44), and 90 people follow the modified Erlang distribution with the parameters (3, 0.14). The reason for setting n = 3 will be explained in the next section. The orange line below represents the line from Figure 2 in reference [1], and the blue line represents the line from the modified Erlang distribution.

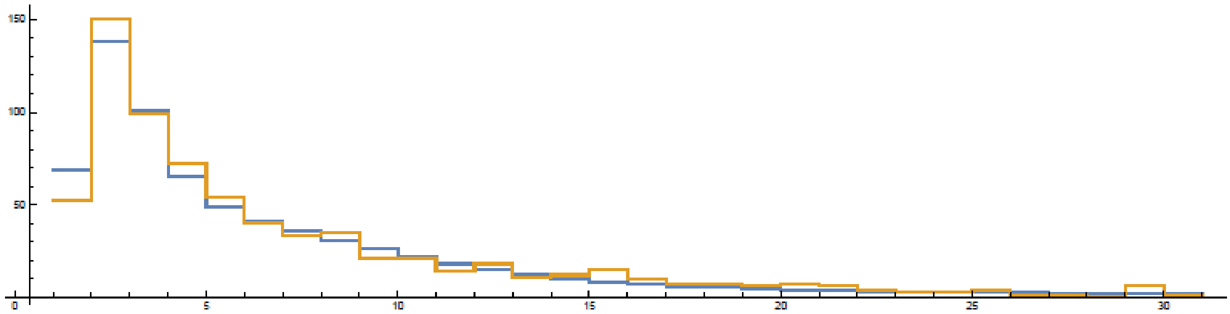


Figure 2:
Comparison of the vaccination model described above with Figure 2 in reference [1], $1 \leq k \leq 30$

In Figure 2 in reference [1], it can be seen that similar graphs are obtained when only the data reported on May 26, June 9, June 23, July 7 and July 21 are used, in which the peak appears on the second or third day and then decreases. It is reasonable to suspect that $\lambda = 1.6$, the rate at which vaccine-induced changes occur, is too high. However, this is because the histogram Figure 2 described in reference [1] was originally provided based on the "Summary of Events Reported as Death after COVID-19 Vaccination" published by MHLW, and it is inevitable that the population had a high "rate of small changes caused by the vaccination." In Figure 2 in reference [1], there is a possibility that some recipients were recorded as "dead on the second day after vaccination if death was confirmed the morning after the day of vaccination". Furthermore, if the vaccine was administered in the evening and the recipient died the next morning, less than 24 hours had passed since vaccination. Therefore, there is a large difference between deaths recorded on the first day and the second day.

3 Reason for setting $n = 3$ and its meaning

This section explains that when n is large, short-term mortality within 5 days is quite different in shape from Figure 2 in reference [1]. To do this, we first consider the mode ρ of the Erlang distribution. Based on Figure 2 in reference [1], the ρ should satisfy:

$$1 < \rho := \frac{n-1}{\lambda} < 3.$$

If $n \geq 2$, the variance of the Erlang distribution is:

$$\frac{n}{\lambda^2} = n\frac{\rho^2}{(n-1)^2} < \frac{9n}{(n-1)^2}.$$

This implies that when n is large enough, the variance of the Erlang distribution is very small. The modified Erlang distribution is thought to be similar. However, Figure 2 in reference [1] cannot be approximated unless the distribution has a relatively large variance. This argument is more effective when ρ is close to 1 because the variance of the Erlang distribution is small. Conversely, if ρ is close to 3, we discuss as follows: when ρ satisfies the above condition, the number of deaths up to day $\rho/3$ is bounded by

$$\int_0^{\rho/3} \frac{\lambda^n}{(n-1)!} x^{n-1} e^{-\lambda x} dx \leq \int_0^{\rho/3} \frac{\lambda^n x^{n-1}}{(n-1)!} dx = \frac{1}{n!}\frac{(\lambda\rho)^n}{3^n}$$

from $e^{-x} \leq 1$ for $x \geq 0$ and the probability density function of the Erlang distribution. Applying Stirling's formula, which is given by

$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \left(1 + \frac{1}{12n} + \frac{1}{288n^2} - \frac{139}{51840n^3} - \frac{571}{2488320n^4} + O(n^{-5})\right)$$

(see [6, Speed of convergence and error estimates]), to $(n-1)!$, we have

$$\frac{1}{n!}\frac{(\lambda\rho)^n}{3^n} = \frac{(n-1)^n}{n3^n}\frac{1}{\sqrt{2\pi(n-1)}}\left(\frac{e}{n-1}\right)^{n-1}\left(1 + O(n^{-1})\right) = \frac{\sqrt{n-1}}{ne\sqrt{2\pi}}\left(\frac{e}{3}\right)^n \left(1 + O(n^{-1})\right).$$

Thus, when n is sufficiently large, the above value becomes very small by

$$e = 2.718281828459 \ldots < 3, \qquad \frac{1}{e\sqrt{2\pi}} = 0.1467626632 \ldots.$$

Therefore, it is not necessary to consider when n is large enough because the number of deaths by day $\rho/3$ is too small when n is large and $1 < \rho < 3$. By gradually decreasing n from $n = 10$, etc., it can be understood by numerical computation that $n = 3$ is optimal. For example, for $n = 7,6,5$, we obtain the following graphs.

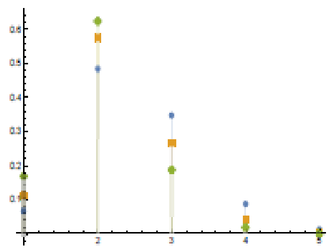


Figure 3: $n = 7$, $p = 3.5, 4, 4.5$

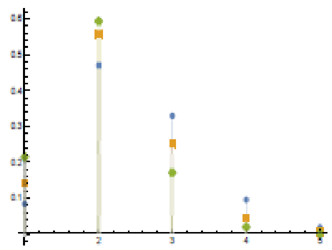


Figure 4: $n = 6$, $p = 3, 3.5, 4$

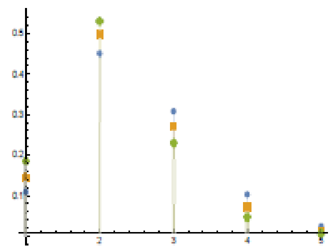


Figure 5: $n = 5$, $p = 2.5, 2.75, 3$

Considering the balance of the height differences on the first, second, and third days, it seems that Figure 2 in reference [1] cannot be approximated by these figures. Therefore, $n = 2,3,4$ are candidates, but here, $n = 3$ is taken as the median value.

In the preceding discussion, it has been assumed that death occurs after a change occurring n times, but the above discussion has shown that the optimal value of n is 3. That is, when dealing with short-term deaths within 5 days, it would be expected that n may be greater than 3, but not greater than 10. This has significant implications: death from three changes means a dramatic worsening of the condition, which suggests sudden death. In fact, the 28-year-old man described in reference [2] died suddenly of myocardial rhabdomyolysis 5 days after the second vaccination. Therefore, this mathematical model is highly rational.

4 Excess mortality

It should be noted that, unlike the previous discussion, this section is not based on specific data from reference [1]. The purpose of this section is to discuss what would happen to excess mortality if the modified Erlang distribution model were correct.

In the previous section, we considered deaths within a month. Now we consider deaths within a year. Therefore, let $n = 3$ and $\lambda = 0.02$ in the modified Erlang distribution. Then the mortality rate is as follows:

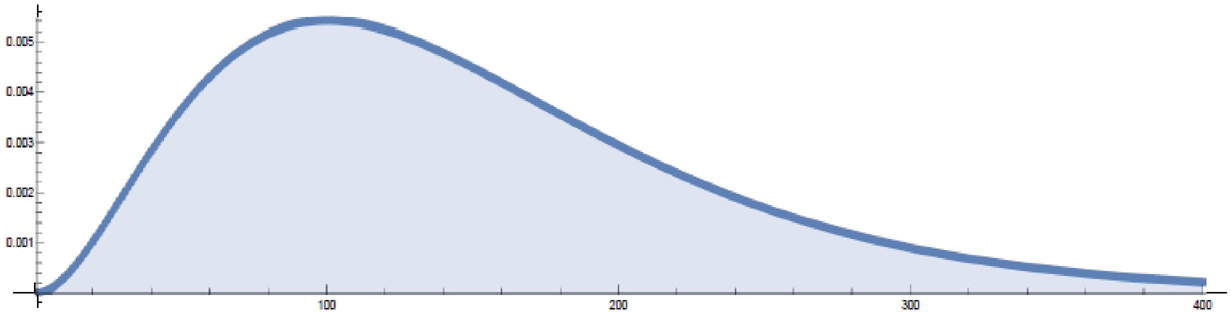


Figure 6: Modified Erlang distribution with parameters (3, 0.02), $1 \leq k \leq 400$

This graph shows that even if $\lambda = 0.02$, which is very high, the mortality peaks around 100 days after vaccination. This means that even if a person is safe for a few days after vaccination, it is not possible to rule out the possibility of death later. Also, because a long time has passed since vaccination, the causal relationship between death and vaccination may not even be discussed.

As before, let $n = 3$ and $\lambda = 0.001$, and draw a graph of the cumulative distribution function (CDF) of the modified Erlang distribution with the parameters (3, 0.001). This graph can then be considered to show by what date and at what rate people die in total.

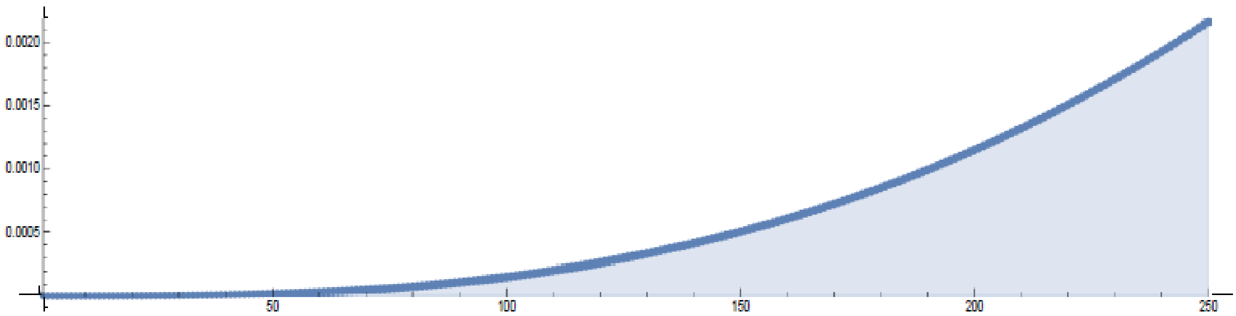


Figure 7: CDF of the modified Erlang distribution with parameters (3, 0.001), $1 \leq k \leq 250$

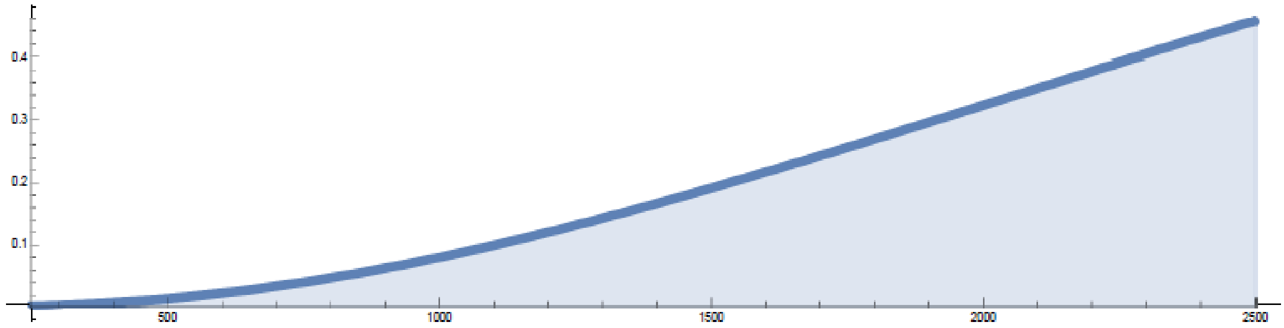


Figure 8: CDF of the modified Erlang distribution with parameters (3, 0.001), $250 \leq k \leq 2500$

These graphs show that, even if $\lambda = 0.001$, which is considerably high, less than 0.26% of vaccinated people will die after about 8 months (Figure 7), but more than 40% of vaccinated people will die after about 2500 days (Figure 8). If the death occurs a considerable period of time after vaccination, $n = 3$ may not hold true, so let $n = 100$ and $\lambda = 0.05$ and draw a graph of the cumulative distribution function of the modified Erlang distribution with the parameters (100, 0.05).

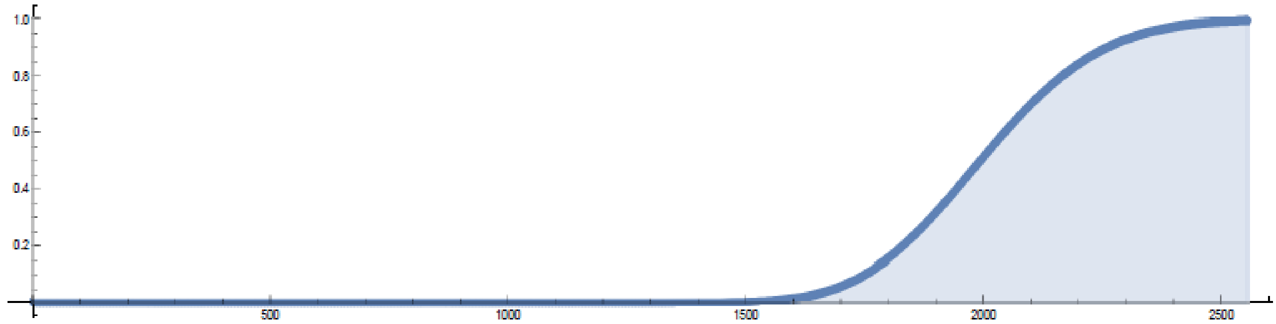


Figure 9: CDF for modified Erlang distribution with parameters (100, 0.05), $1 \leq k \leq 2555$

After about 50 months, the total number of deaths is almost 0, but after about 60 months, deaths gradually become apparent, and after about 2500 days, that is, about 7 years, more than 95% of vaccinated people have died (Figure 9). These figures show that in the model with a small n, each individual dies suddenly, but as a population the deaths occur slowly, while in the model with a large n, the deaths occur suddenly as a population, but each individual dies slowly. At present, the possibility that this mathematical model is associated with the drastic increase in excess mortality observed throughout the world in 2022 cannot be ruled out. Furthermore, these mathematical experiments raise the question of whether approval of the vaccines in less than 8 months from the start of a trial was correct.

Conclusion

This article suggests that the histogram described in Figure 2, reference [1], can be approximated using the modified Erlang distribution by appropriately dividing it into short-term mortality, medium-term mortality, and long-term mortality. It is natural to think that very long-term deaths, which may be related to excess mortality, can also be explained by changing the parameters of the Erlang distribution. The proposal that patterns of post-vaccination mortality and excess mortality can be interpreted using the same mathematical principles may provide important guidance in collecting data on both.

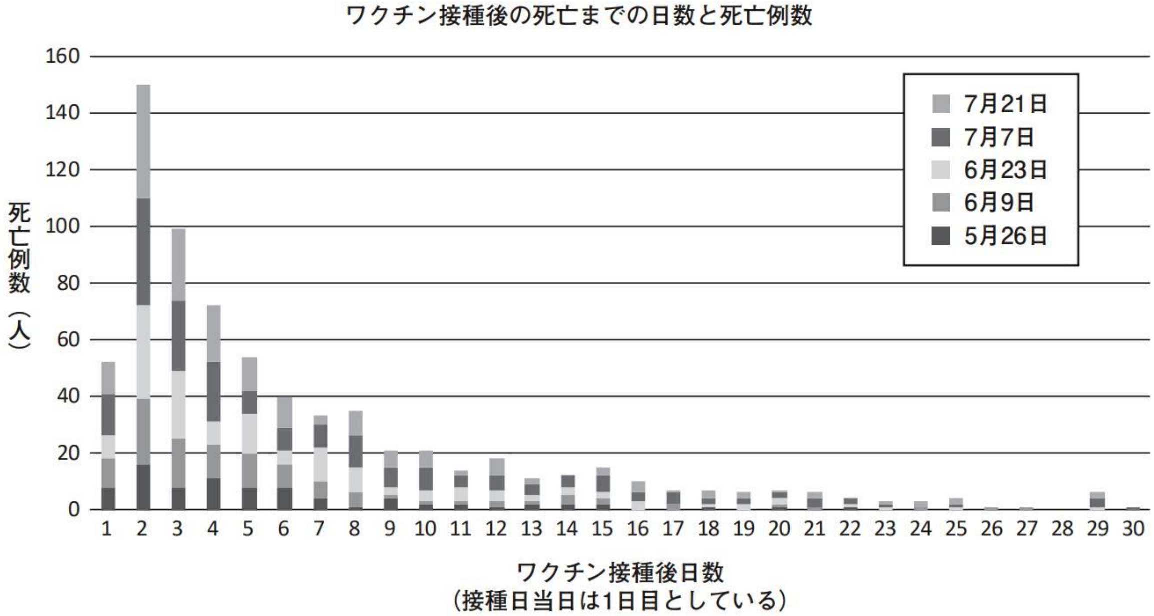
Acknowledgement

I would like to express my gratitude to Dr. Masanori Fukushima, Mr. Takayuki Kikuchi, Dr. Eiji Nakatani, and Dr. Tsutomu Nishimura for their very useful advice and comments in the Zoom meeting.

Reference data: Histogram described in Ref. [1] Figure 2

The vertical axis is the number of deaths, and the horizontal axis is the number of days after vaccination (day 1 refers to the day of vaccination).

Fig. 2 Number of reported deaths on each day after vaccination



References

[1] Masanori Fukushima, Yuriko Hirai, Eiji Nakatani and Tsutomu Nishimura, Overview of COVID-19 postvaccination mortality and pharmacoepidemiological evaluation: nation-wide view and a proposal, Clinical Evaluation. 2022;49(3):499-517.

[2] Masanori Fukushima, Takayuki Kikuchi and Yuriko Hirai, Warnings and Requests to Coronavirus Vaccine Recipients and All Healthcare Providers -Based on the case of a healthy 28-year-old man who died suddenly of myocardial rhabdomyolysis 5 days after vaccination with the novel coronavirus vaccine, Clinical Evaluation. 2023;50(4):507-542.

[3] Kiyoshi Ito, [Probability theory and I], Kakuritsuron to watashi (in Japanese), Iwanami Shoten, Publishers, (2010/9/15) ISBN 9784000052085.

[4] Wikipedia, Erlang distribution, https://en.wikipedia.org/wiki/Erlang_distribution

[5] Wikipedia, Exponential distribution, https://en.wikipedia.org/wiki/Exponential_distribution

[6] Wikipedia, Stirling's approximation, https://en.wikipedia.org/wiki/Stirling's_approximation