Geometric Origin of Leptonic CP Violation from Oriented Internal Area

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Abstract

We propose a geometric origin of leptonic CP violation based on complexified distance-exponential kernel matrices. When internal coordinates are extended to two dimensions, a natural oriented area term generates complex phases in the mass matrix. We show that the Jarlskog-type CP invariant is directly proportional to the oriented area of the internal triangle. CP conservation corresponds to collinearity in internal space, while maximal CP violation emerges for maximal internal area.

Keywords
CP violation, neutrino mixing, geometric mass generation, Jarlskog invariant, internal space

1 Introduction

The origin of leptonic CP violation remains an open question. Experimental indications suggest a large CP phase in the neutrino sector [1]. We investigate whether CP violation may arise geometrically from the orientation of internal coordinates in distance-dependent kernel mass matrices.

2 Complexified Distance Kernel

We extend the real distance kernel to

$$M_{ab} = m_0 \exp\!\left(-A|\alpha_a - \alpha_b|^2 + iB\,\Omega_{ab}\right), \tag{1}$$

where $\alpha_a \in \mathbb{C}$ and

$$\Omega_{ab} = \mathrm{Im}(\alpha_a \alpha_b^*). \tag{2}$$

In real coordinates $\alpha_a = x_a + iy_a$,

$$\Omega_{ab} = x_a y_b - y_a x_b, \tag{3}$$

which represents an oriented area element.
The matrix satisfies

$$M_{ab} = M_{ba}^*, \tag{4}$$

and is therefore Hermitian.

3 Geometric CP Invariant

CP violation is characterized by a Jarlskog-type invariant [2]

$$J = \mathrm{Im}(M_{12} M_{23} M_{31}). \tag{5}$$

Substituting the kernel form yields

$$J = |M_{12}||M_{23}||M_{31}| \sin(\phi_{12} + \phi_{23} + \phi_{31}), \tag{6}$$

with

$$\phi_{ab} = B\,\Omega_{ab}. \tag{7}$$

Using the identity

$$\Omega_{12} + \Omega_{23} + \Omega_{31} = 2\mathcal{A}_\triangle, \tag{8}$$

where $\mathcal{A}_\triangle$ is the oriented area of the triangle formed by $\alpha_1, \alpha_2, \alpha_3$, we obtain

$$J \propto \sin(2B\mathcal{A}_\triangle). \tag{9}$$

4 Geometric Interpretation

We observe:

- If the three internal points are collinear, $\mathcal{A}_{\triangle} = 0$ and CP is conserved.
- If the triangle has maximal area, CP violation becomes maximal.

Thus leptonic CP violation is equivalent to non-zero oriented internal area.

5 Conclusion

We demonstrated that CP violation in distance-exponential kernel matrices arises geometrically from the oriented area of internal coordinates. The CP invariant is directly proportional to this area. This suggests that mass hierarchy, mixing angles, and CP violation may share a unified geometric origin.

References

[1] Particle Data Group, "Review of Particle Physics," Prog. Theor. Exp. Phys. (2024).

[2] C. Jarlskog, "Commutator of the quark mass matrices in the standard electroweak model," Phys. Rev. Lett. 55, 1039 (1985).