The velocity center theorem of a spinning mobile object and its application to the vehicles

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Abstract

In this paper, focus on the mathematical physical theorem of spinning mobile objects, and introduce its application to the vehicles. By using this theorem, and considering the velocity center point CV of the rotating moving object and the turning center point CT of the representative point of the object separately, it will be able to explain various vehicles motion with one simple concept. In addition, it will be possible to clearly explain that the intersection point of the wheels axes of an automobile does not necessarily become the turning center point. This concept can also be applied to improving the driving stability of vehicles and to making quick steering decisions in autonomous driving. Since this theorem can be widely applied to various mobile objects, it is expected to be applied to new fields other than vehicle using wheels.

Keywords : velocity center, speed center,Omnidirectional, rotating , spinning, vehicle, mobile object, mobility

1. Introduction

As the conventional mobility using wheels, in addition to automobiles, the omnidirectional vehicle like Schneider propeller type[1] and like carriage type[2], etc. have been proposed. But there is nothing that can explain all these movements in one concept.

In the case of automobiles, it is often thought that the intersection point of the four wheels axes is the turning center point. But, if the steering angle is changed dynamically, the intersection point of the wheels axes is not always the turning center point. In this regard, there is no specific and systematic description.

In this paper, introduce the one simple theorem about spinning mobile objects, and its application to mobility. Using this theorem, it will be able to explain various vehicle motion with a single concept. Also, it becomes clear that the intersection point of wheels axes does not necessarily become the turning center point.

Specifically,

Chapter 2 describes about theorems concerning spinning mobile object,

Chapter 3 discusses the case which the theorem is applied to automobiles,

Chapter 4 discusses the case which the theorem is applied to a two-wheel drive steering omnidirectional vehicle,

Chapter 5 introduces about an example of an actual prototype.

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2. The velocity center theorem of the spinning mobile object

2.1 The theorem about Absolute velocity $\overrightarrow{V_N}$ of any mass point N on the spinning mobile object U

On the same plane, when the representative point CS on an object U moves velocity $\overrightarrow{V_{CS}}$ with spinning angular velocity Ω_{cs},and with expansion/ contraction,

Absolute velocity $\overrightarrow{V_N}$ at any point N on the object U is expressed as follows,

$$\overrightarrow{V_N} = \sqrt{\varepsilon^2 + {\Omega_{cs}}^2} Rot\left(\frac{\pi}{2} - \alpha\right) \overrightarrow{CV\text{-}N}$$

At this time、ε、α、$Rot(\theta)$ and other condition are as follows.

- Point CS on the object U, mass point N, and velocity $\overrightarrow{V_{CS}}$ of point CS are coplanar
- ε is proportionality constant of expands and contracts. An object U expands and contracts at a speed of εL_{CS-N}. L_{CS-N} is the distance between point CS and point N.
- $Rot(\theta)$ is the rotation matrix that rotate θrad on the same plane S
- α is a value that satisfies the following conditions.

  $$\sin(\alpha) = \varepsilon / \sqrt{\varepsilon^2 + {\Omega_{cs}}^2}\ ,\ \cos(\alpha) = \Omega_{cs} / \sqrt{\varepsilon^2 + {\Omega_{cs}}^2}$$

- The point CV is the position of the distance $\left|\overrightarrow{V_{CS}}\right| / \sqrt{\varepsilon^2 + {\Omega_{cs}}^2}$ from point CS,

  And the direction of $\overrightarrow{CS\text{-}CV}$ is the direction from $\overrightarrow{V_{CS}}$ to $\pi/2$+α rad
- $\overrightarrow{CV\text{-}N}$ is the vector from the point CV to the point N.

Therefore, the speed $\left|\overrightarrow{V_N}\right|$ is $\sqrt{\varepsilon^2 + {\Omega_{cs}}^2}\left|\overrightarrow{CV\text{-}N}\right|$ ($\left|\overrightarrow{CV\text{-}N}\right|$ is the distance from point CV to point N.),

And the direction of $\overrightarrow{V_N}$ is the direction from $\overrightarrow{CV\text{-}N}$ to π/2 -α rad.

Like this, the speed of point N is determined by the distance from point CV to N.

So, point CV defines as the velocity center point, and this theorem define as the velocity center point theorem of the spinning mobile object. Generally, the velocity center point CV doesn't have to be on the rigid body U.

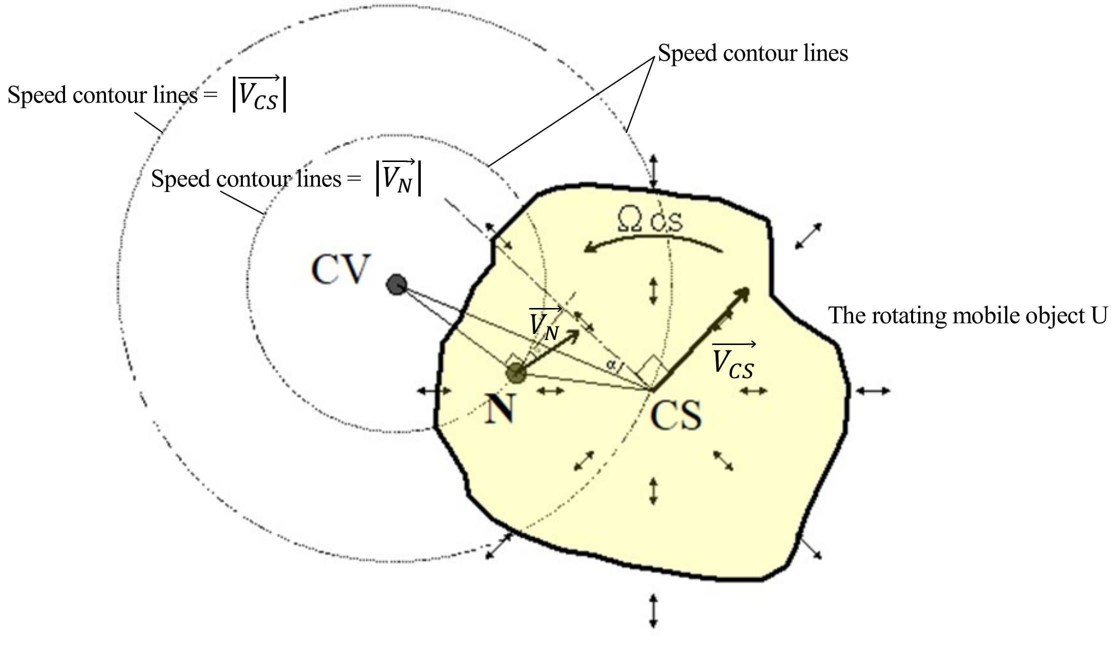


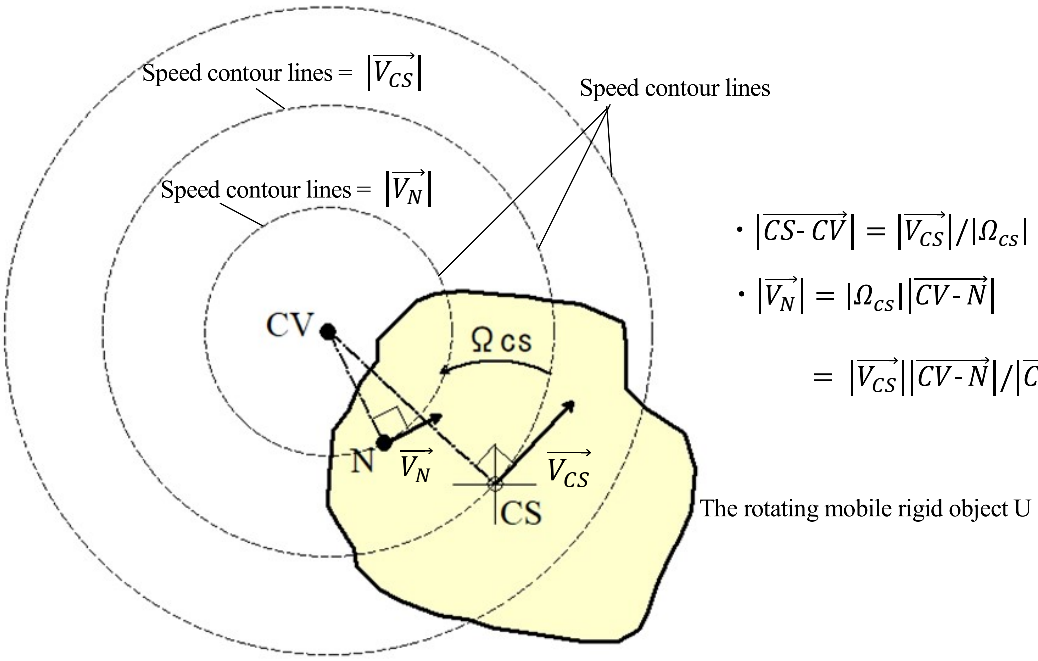
Fig. 2.1-1 The relationship between CS, CV and N of the rotating mobile object that is expanding or contracting

2.2 The theorem about Absolute velocity $\overrightarrow{V_N}$ of any mass point N on the spinning mobile rigid object U

Especially, in case rigid objects, corresponds to ε = 0 in chapter 2.1.
The point CV is the position of the distance $|\overrightarrow{V_{CS}}|/|\Omega_{cs}|$ from point CS,
And its direction is the direction from $\overrightarrow{V_{CS}}$ to $\pi/2$ rad (When $\Omega_{cs} > 0$, α = 0) or $-\pi/2$ rad (When $\Omega_{cs} < 0$, α =π).
So, the absolute velocity $\overrightarrow{V_N}$ at any point N on the rigid object U is expressed as follows,

$$\overrightarrow{V_N} = \Omega_{cs}\, Rot\left(\frac{\pi}{2}\right) \overrightarrow{CV\text{-}N}$$

Therefore, the speed $|\overrightarrow{V_N}|$ is $|\Omega_{cs}||\overrightarrow{CV\text{-}N}|$,
And the direction of $\overrightarrow{V_N}$ is the direction from $\overrightarrow{CV\text{-}N}$ to $\pi/2$ rad (When $\Omega_{cs} > 0$) or $-\pi/2$ rad (When $\Omega_{cs} < 0$).



- $|\overrightarrow{CS\text{-}CV}| = |\overrightarrow{V_{CS}}|/|\Omega_{cs}|$
- $|\overrightarrow{V_N}| = |\Omega_{cs}||\overrightarrow{CV\text{-}N}|$

$$= |\overrightarrow{V_{CS}}||\overrightarrow{CV\text{-}N}|/|\overrightarrow{CS\text{-}CV}|$$

Fig.2.2-1 The relationship between CS, CV and N of the rotating mobile rigid

In this paper, introduce the application of this theorem to mobility.

3. The relationship between this theorem and the motor vehicle movement

In this chapter, introduce the case that the velocity center theorem of chapter 2.2 is applied to the motor vehicle movement.

3.1 The rolling motion of the wheel

As shown in Figure.3.1-1(A), in the case of the wheel rolling on the flat plane, the velocity center point CV is the contact point between the wheel and the flat plane.

IF wheel's radius is R and the rolling angular velocity is Ω_{CS},

the moving speed of the velocity center point CV is $R\ \Omega_{CS}(=|\overrightarrow{V_{CV}}|)$.

From the velocity center theorem of the chapter2.2, in the case of any mass point N which is on the wheel,

$|\overrightarrow{V_N}|$ is proportional to the distance from the point CV to N $(|\overrightarrow{V_N}| = |\Omega_{cs}||\overrightarrow{CV\text{-}N}|)$,

and $\overrightarrow{V_N}$ and $\overrightarrow{CV\text{-}N}$ are always orthogonal.

So, if point CV and point N overlap, $|\overrightarrow{CV\text{-}N}| = 0$, therefore $|\overrightarrow{V_N}| = 0$. However, as mentioned before, $|\overrightarrow{V_{CV}}| = R\ \Omega_{CS}$.

And the rolling motion of the wheel is represented by speed contour lines like figure3.1-1(B),

and the two states(A)(B) in the figure 3.1-1 are equivalent.

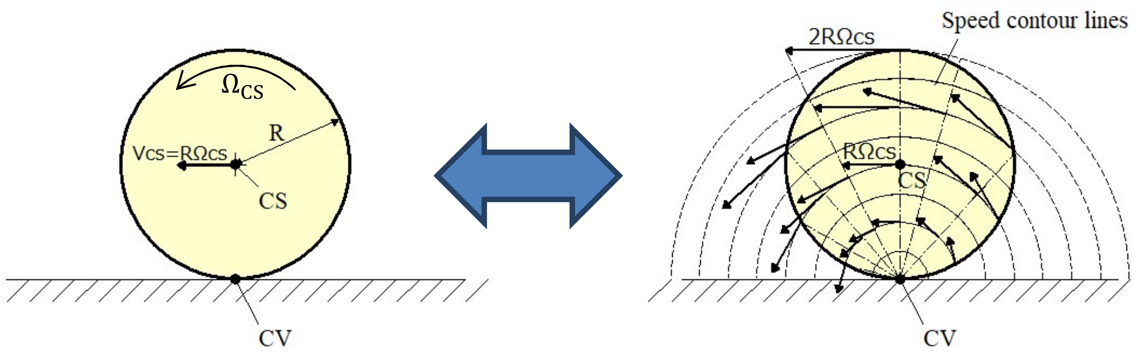
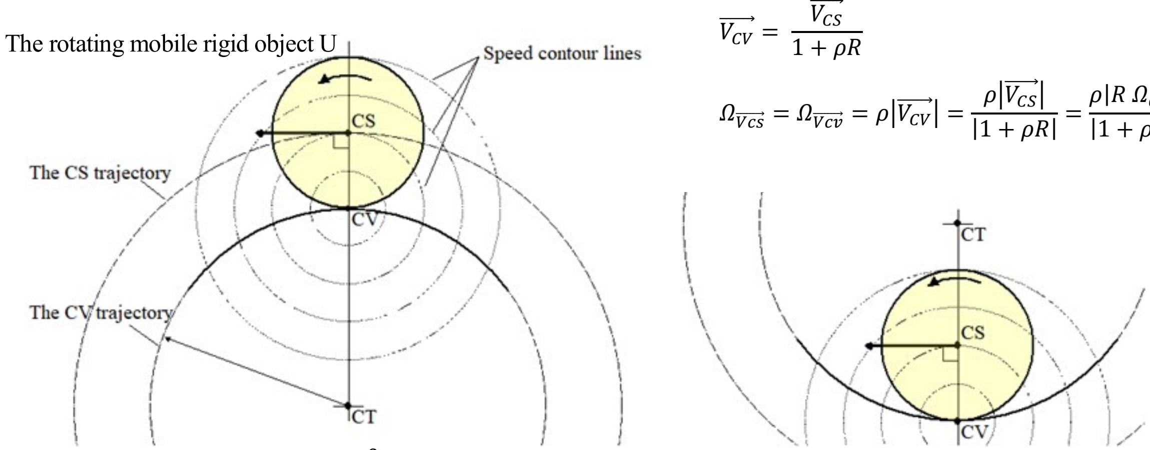


Fig.3.1-1 The relationships between point CV and the rolling object on the flat surface

The figure3.1-2 shows an image of a wheel rolling on a curved surface with the center of curvature at point CT.

In this case, the turning center of point CV and point CS are point CT, and point CV and point CT do not coincide.

Next chapter, introduce an example where point CV and point CT coincide.



$$\overrightarrow{V_{CV}} = \frac{\overrightarrow{V_{CS}}}{1+\rho R}$$

$$\Omega_{\overrightarrow{Vcs}} = \Omega_{\overrightarrow{Vcv}} = \rho|\overrightarrow{V_{CV}}| = \frac{\rho|\overrightarrow{V_{CS}}|}{|1+\rho R|} = \frac{\rho|R\ \Omega_{CS}|}{|1+\rho R|}$$

$\rho > 0$ $\rho < 0$

Fig.3.1-2 The relationships between CS,CV,CT and the rolling object on the curved

3.2 The turning motion of motor vehicle

When turning while keeping a constant wheel angle in relation to the vehicle body,
The velocity center points CV and the turning center point CT coincide and neither point moves. (Fig.3.2-1)

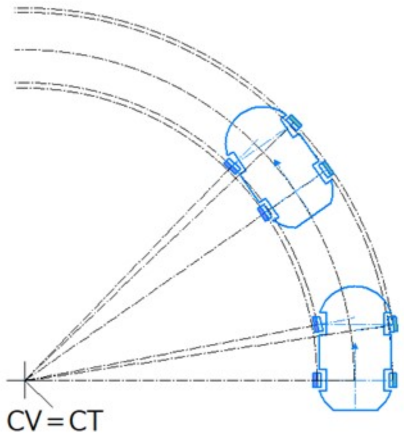


Fig.3.2-1 The relationships between CV and CT of the automobile

Also, as shown in Fig.3.2-2,
consider a circle centered on point CS and passing through the wheel (point N), If the tangent direction of the circle at point N is angle 0 rad, the steering angle ϕ of the wheel (point N) is equal to the angle ∠CS-N-CV.

If the angular velocity of the vehicle is $\overrightarrow{\Omega_{CS}}$, and the angular velocity of point CS's velocity $\overrightarrow{V_{CS}}$ is $\Omega_{\overrightarrow{V_{CS}}}$,

Point N moves around point CS at angular velocity $\overrightarrow{\Omega_{CS}} - \overrightarrow{\Omega_{\overrightarrow{V_{CS}}}}$.

The steering angle ϕ rad changes according to the movement of point N as seen from point CS.

In the case of Fig.3.2-1, $\overrightarrow{\Omega_{CS}} = \overrightarrow{\Omega_{\overrightarrow{V_{CS}}}}$, so, the angular velocity of point N as seen from point CS is 0,

therefore the value of angle ∠CS-N-CV remains the same, as a result, the steering angle ϕ is constant.
The appropriate position of point CS on the vehicle depends on the design concept and the situation.
It will be discussed in the following chapters.

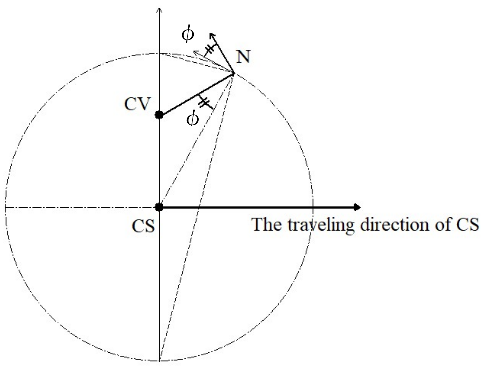


Fig.3.2-2 The relationships between the steering angle and CS, CV

3.3 The motor vehicle movement that the velocity center point CV and the turning center point CT don't coincide

When the moving direction of point CS is in the θ rad direction relative to the vehicle center line,
set the vehicle center point CV as shown in Fig.3.3-1(A),
and set the wheel's direction and its speed to satisfy the velocity center theorem of chapter2.2.
In the case of a motor vehicle, the intersection of the wheel axes is often said the turning center point.
But point CV becomes the turning center point CT only when the steering angles are constant as shown in Fig.3.2-1.
For example, if the representative point CS moves on a straight line as shown Fig.3.3-1(A), while the direction of travel of point CS remains constant, and θ is continuously changed to satisfy the velocity center theorem of chapter2.2. In this way, point CS goes straight ahead, while the vehicle body turns in the direction that approaches the posture shown in Fig.3.3-1(B). At this, the point CS moves straight, but there is no turning center point. Thus, the velocity center point CV and the turning center point CT don't coincide.
The representative point CS may be placed anywhere on the car body. For example, it can be taken to the edge of the vehicle as shown in Fig.3.3-1(C). As with (A), if taking a point CV satisfying the velocity center theorem of chapter2.2, be able to move the point CS in the direction of the arrow. If setting a point CS where it is likely to collide, it is useful when moving in a direction that avoids collision.
The movements described in this chapter may be performed unconsciously by skilled drivers, but using this method, objective explanations are possible.
Also, the range of θ is determined by the maximum steering angle of the front wheels. But the range of θ is larger when point CS is positioned forward than when point CS is positioned rearward. This is one of the reasons why many people are not good at driving in reverse. Where point CS is appropriate depends on the situation at the time. If the target of the orbit of point CS is not a straight line, but an arc, point CV moves closer to the center point of the arc (point CT).

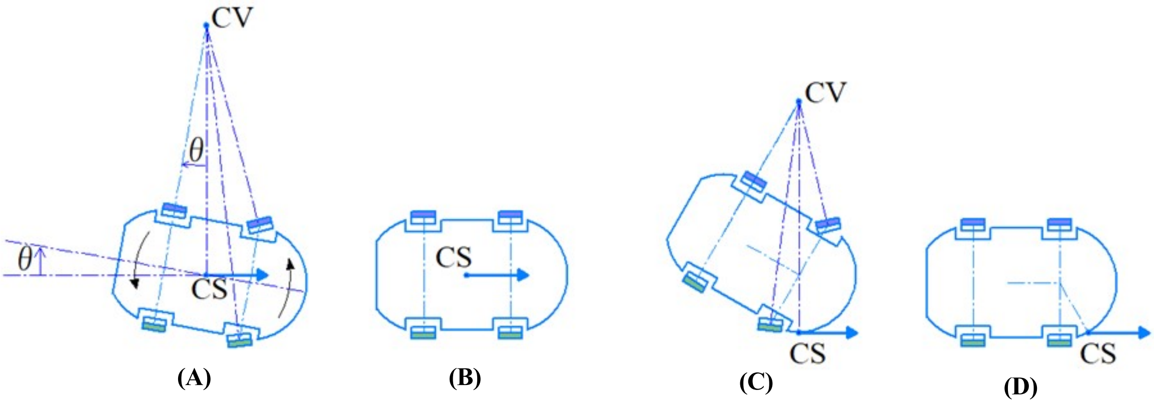
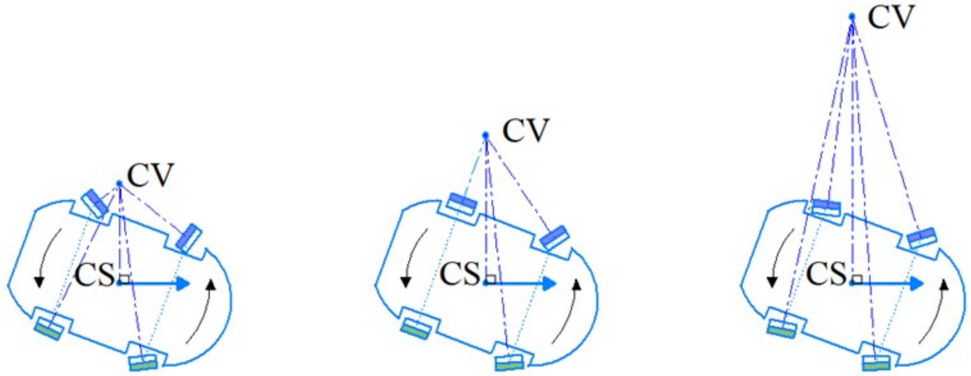


Fig.3.3-1 The relationships between CV and CS of the vehicle

3.4 Four-wheel steering movement

If the concept of the previous chapter is applied to the four-wheel steering movement, it is shown as Fig.3.4-1.
With respect to point CV of (B),
the rear wheels are steered in reverse phase like(A) when point CV is close to point CS,
and in same phase like (C) when point CV is far away from CS.
Movements that was impossible with two-wheel steering vehicle, such as intentionally swing the vehicle body while the representative point CS is going straight, will become possible with four-wheel steering vehicle.
Conversely, it is possible to make the vehicle more stable. The steering function is not just a function for curving, but it can become a more important function for improving driving stability and attitude control.



(A) Reverse phase **(B)** **(C) Same phase**

Fig.3.4-1 The relationships between the steering angles and CV.CS of the four-wheel-steering vehicle

4. Example of application to Omnidirectional vehicle, etc.

In this chapter, using two-wheel steering vehicle like Fig.4-1, and show that omnidirectional vehicle, etc can be realized.

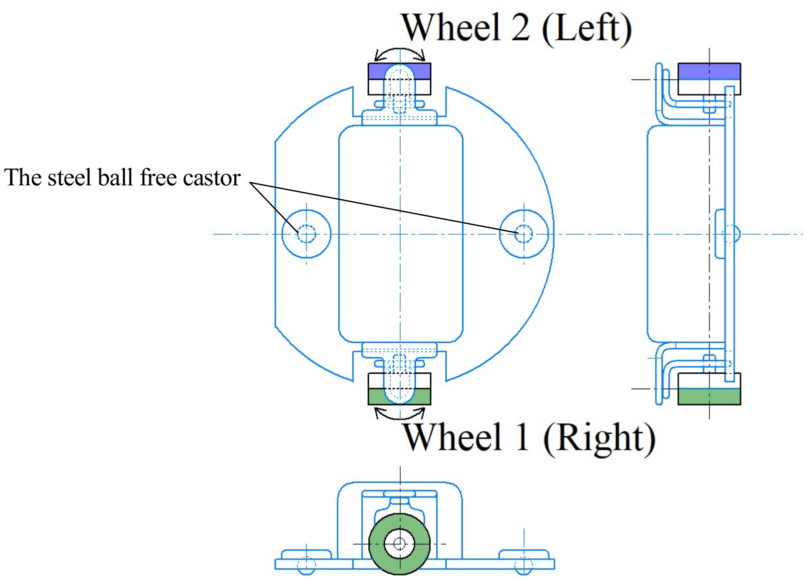


Fig4-1 The two-wheel-drive and steering mobility model

In order to achieve more flexible motion, it is necessary to control the positional relationship between point CV and point CS appropriately.

The angular velocity vector $\overrightarrow{\Omega_{\overrightarrow{V_N}}}$ of the absolute velocity $\overrightarrow{V_N}$ of any point N on the rotational object is expressed as follows,

$$\overrightarrow{\Omega_{\overrightarrow{V_N}}} = \Omega_{cs}\,\overrightarrow{e_N} - \frac{1}{\left|\overrightarrow{CV\text{-}N}\right|^2}\overrightarrow{CV\text{-}N} \times \overrightarrow{V_{CV}}$$

$\overrightarrow{e_N}$ is the unit vector in the direction of the cross product of $\overrightarrow{CV\text{-}N}$ and $\overrightarrow{V_N}$,

Therefore,

If the wheel position is point N, and when point N approaches point CV, care must be taken not to make $\left|\overrightarrow{\Omega_{\overrightarrow{V_N}}}\right|$ too large.

But, For mobility, it may be useful to move the point CV appropriately, so that the limit value of $\left|\overrightarrow{\Omega_{\overrightarrow{V_N}}}\right|$ when point N approaches point CV does not diverge.
Such examples are described in Chapter4.2, 4.3, 5,etc.

4.1 Example 1 of omnidirectional movement (Frisbee type)

Take a velocity center point CV and a representative point CS as shown in Figure 4.1-1, and satisfy the velocity center theorem of chapter2.2, do from ① to ⑧ continuously, while the moving object rotates counter-clockwise, the representative point CS moves in the direction of the arrow (in Figure 4.1-1, it moves to the right).

Figure 4.1-2 is the superimposed view of points CS and CV from ① to ⑧ in Figure 4.1-1.
Figure 4.1-3 shows an example where the representative point CS can go at right angles from A to B to C route with zero stopping time by varying the distance between points CV and CS appropriately.
Even in this case, the velocity center point CV (the intersection of the wheels axes) is not the turning center of point CS.
If the distance between CS and CV is constant, and the CS goes straight, the trajectories of the wheels draw trochoid curves.
The farther a point CV is from a point CS, the longer distance CS travels in one rotation.
Here, the point CS is the center of the two wheels, but the representative point CS can be placed anywhere on the vehicle.

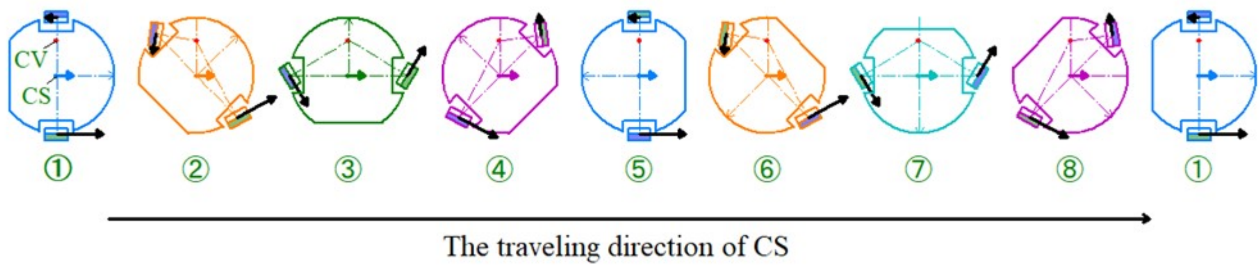


Fig.4.1-1 Frisbee type; The relationships between each posture and wheels angles

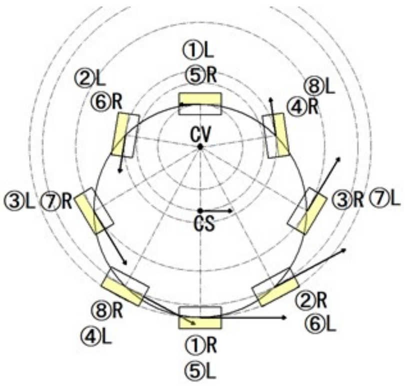


Fig.4.1-2 Layered ①～⑧ of Fig.3.1-1

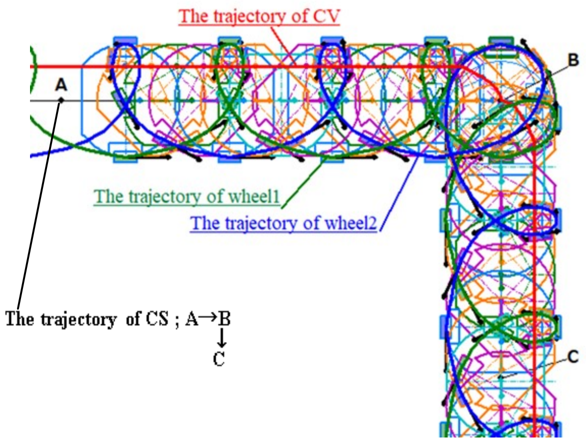


Fig.4.1-3 Frisbee type; The trajectory of CV,CS and wheels

4.2 Example 2 of omnidirectional movement （ No steering wheel type ）

As shown in Fig.4.2-1, point CV is the intersection of the wheel axis and the line passing through point CS which is perpendicular to the direction of travel of point CS. By setting the speed of the wheel to the ratio of the distance from this point CV, point CS can move in any direction without steering the wheel.(Fig.4.2-1and Fig4.2-2 are examples of point CS moving to the right.) The representative point CS may be placed anywhere on the body, except on the wheel axis. (Same as chapter3.3) If the target orbit of point CS is an arc, point CV approaches the turning center point CT of point CS.
(In the case that the trajectory of point CS is a straight line, the curvature ρ → 0, and the radius of the arc is infinity, so the point CV goes away from the point CS infinitely, as shown in Fig.4.2-2.)

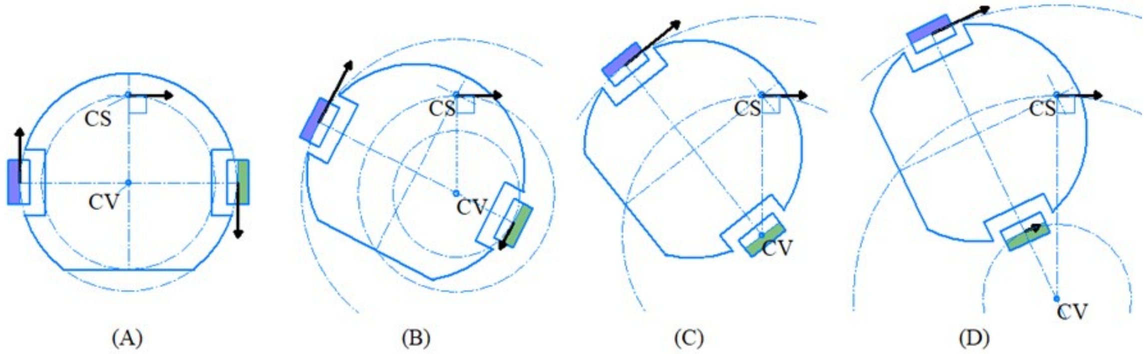


Fig.4.2-1 Non-steering type; The relationships between CS and CV in each posture

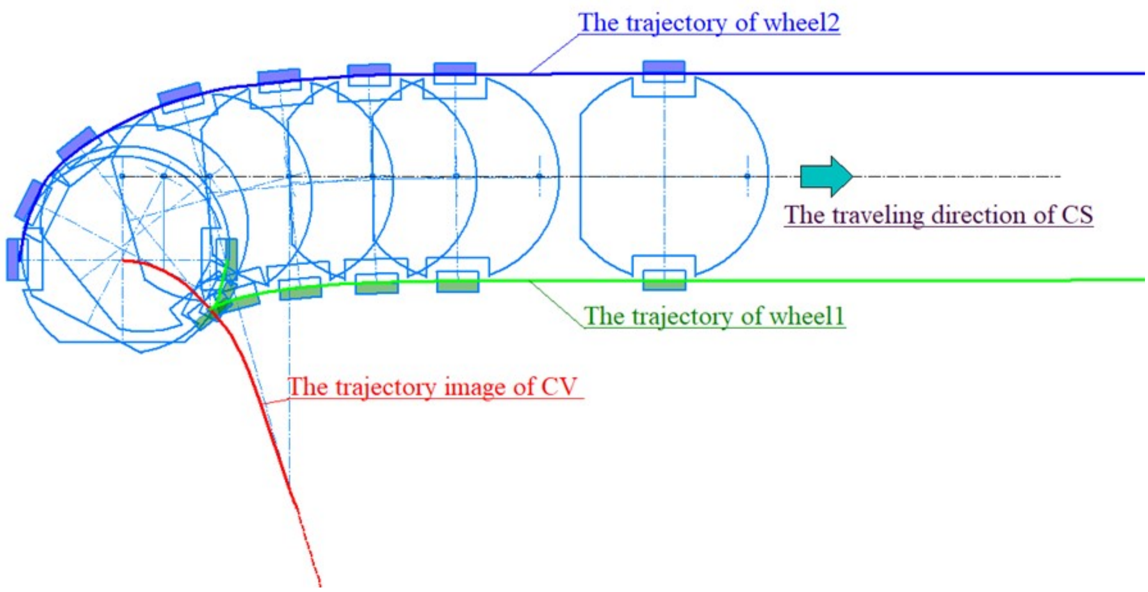


Fig.4.2-2 Non-steering type; The trajectory of CV, CS and wheels

4.3 Example 3 of omnidirectional movement (Non-Frisbee type)

Frisbee type must continue to rotate, No steering wheel type requires the point CS to be away from the drive wheel axis.
In this Chapter, Shows an example where the point CS can be moved in any direction, even if it is on the drive wheel axis and does not 1 to rotate.
For example, if the two-wheel drive steering vehicle is moved as shown from ① to ⑧ in Fig.4.3-1, even if there is point CS on the wheel axis , point CS can be moved in the ξ direction from the center line of the vehicle.
From ① to ⑧ are as follows;
① : Consider the case where a stationary vehicle starts to move to the right as shown in ①. Point CV is at point CS.
② : When point CV is gradually moved away from point CS, point CS moves to the right while the vehicle rotates counterclockwise.

③ : The point CV is moved from ②to ③, and when point CV is at the position ③,, set the point CV to be on the center of the left wheel and the steering angle of the left wheel is ϕ .

④ : After ③, point CV gradually moves away from point CS.

⑤ : Finally, as in ⑤, the distance between point CV and point CS is $+\infty$, so that the direction of travel of point CS and the direction of travel of both wheels are the same.

⑥ : The position of the wheel is the same as ⑤, but assume that point CV is on the opposite side of $-\infty$.

⑦ : Gradually bring point CV closer to point CS. This time, point CS moves to the right while rotating clockwise.
Once the point CV reaches a specific point, move the point CV far away from point CS again,
Adjust so that the final posture is like ⑧ .

⑧ : In the position of ⑧, the speed of both wheels is the same, and the vehicle moves to the right, facing the front.

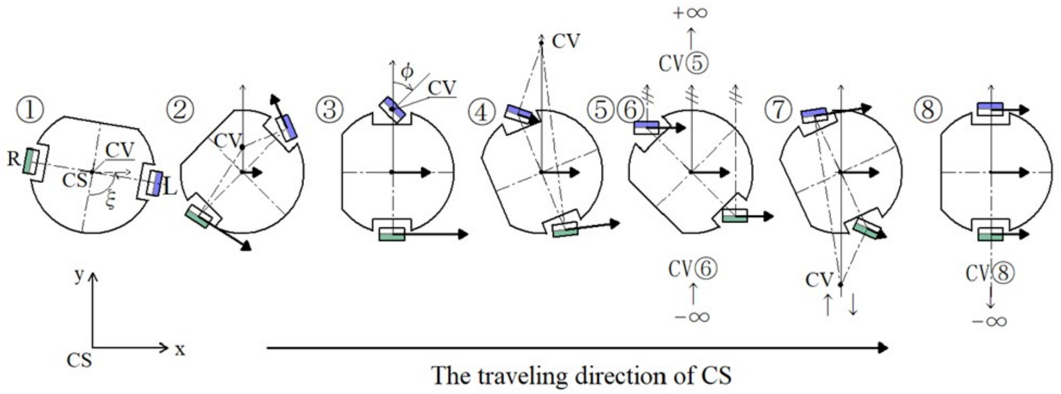


Fig.4.3-1 Non-Frisbee type; The relationships between CV and wheels angles in each posture

Superimposing ① through ⑧ in Fig.4.3-1 results in Fig.4.3-2.

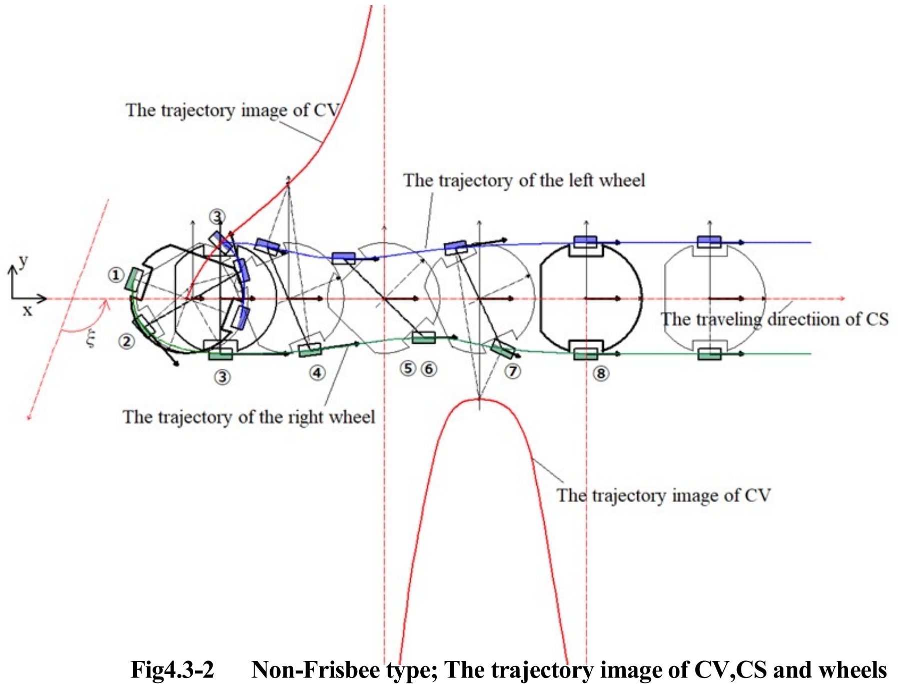


Fig4.3-2 Non-Frisbee type; The trajectory image of CV,CS and wheels

If the target orbit of point CS is an arc which turning center is point CT,
The position of point CV corresponding to Fig.4.3-2⑤⑥ is point CT.
When viewed from point CS, the point CV which corresponding to Fig. 4.3-2④～⑤、⑥～⑦ is before and after the turning center point CT, finally point CV approaches the point CT.
(In the case that the trajectory of point CS is a straight line, the curvature $\rho \rightarrow 0$, and the radius of the arc is infinity, so the point CV is away from the point CS infinitely, as shown in Fig.4.3-2.)

5. Prototype machine

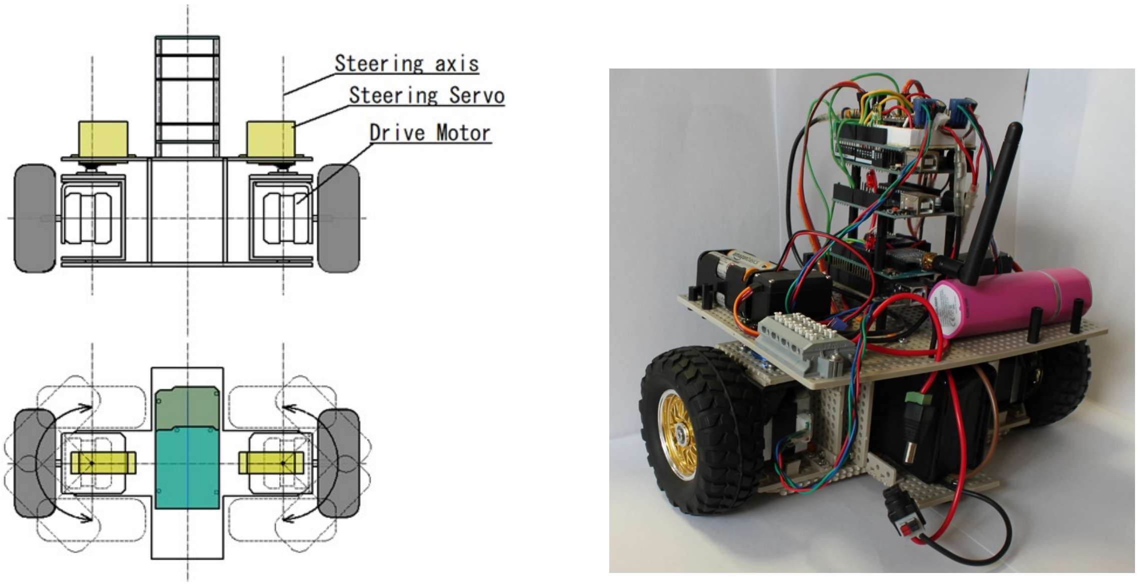
As shown in Fig. 5.1-1, the prototype is a two-wheel drive steering type which steering axis and wheel center of gravity are separated.

The steering axis of the wheel is not a type that passes through the center of gravity of the wheel like the ideal model in Chapter 4. But the basic point of determining the point CV according to the velocity center theorem of chapter2.2 and determining the speed and direction of the wheel is the same.

For example, if the motion shown in chapter4.3 is reproduced on a prototype, Fig.4.3-1 corresponds to Fig.5.1-2.

In Figure 4.3-1③, the speed of the wheel that overlaps point CV is 0. On the other hand, in the prototype, steering axis and wheel center of gravity are separated, so

In Fig. 5.1-2, the speed of the wheel does not become 0 even when the posture is like③a, and when the state is like ③b, the speed of the wheel that overlaps the point CV becomes zero, and the direction of rotation changes before and after that.



(a) The configuration (b) The photograph

Fig5.1-1 The omnidirectional prototype vehicle

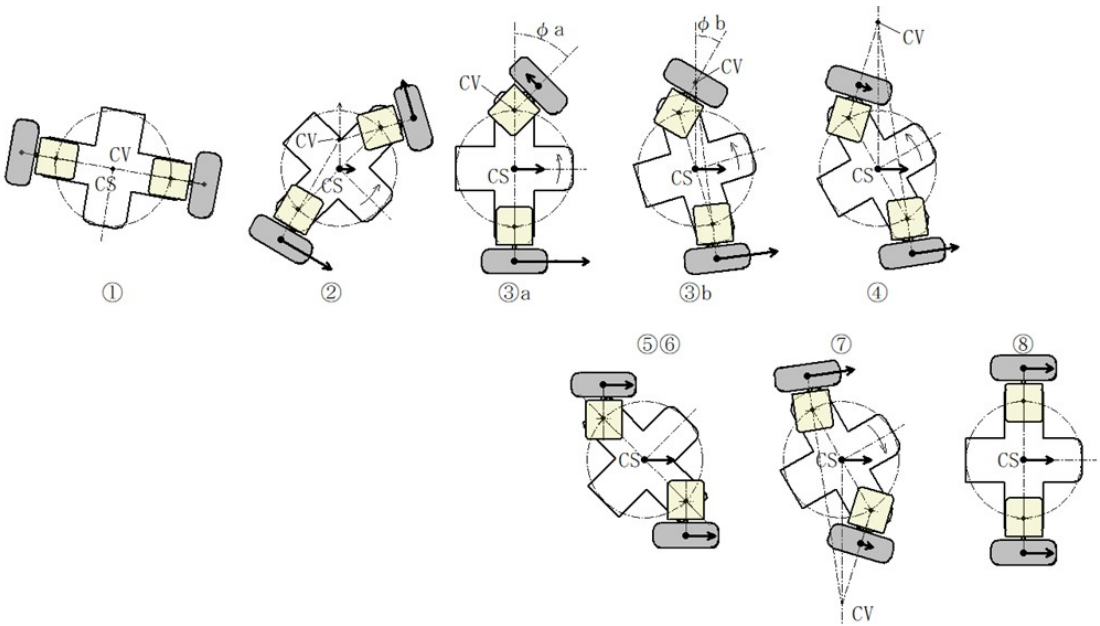


Fig5.1-2 The prototype vehicle; The relationships between CV and wheels angles in each posture

6. Results

By using the theorem in this paper, various applications for wheeled vehicle were demonstrated. In addition, it was shown that the intersection point CV of the wheel axis is not necessarily the turning center.

Because of a simple theorem, it is possible to apply it to improved driving stability, the rapid judgment of avoiding collisions in autonomous driving, etc., and it is thought that it will contribute to the development of new vehicle in the future. Also, since this theorem can be applied not only to vehicle using wheels but also to all moving objects, it is expected to be applied to various vehicle applications.

Finally, as for the proof of the theorem, there are various types of proofs, such as proofs using coordinates, geometric proofs, and proofs using physical intuition, but those are not difficult proofs. So, those proofs omitted in this paper.

7. REFERENCES

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