Boosting the Magnetic Field of a Toroidal Conductive Fluid by a Polodal Flow

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Researchers have struggled to understand the mechanism underlying the formation of celestial magnetic fields. Currently, the generation of axisymmetric and poloidal magnetic fields can be solved by complex convection arguments. There are also simple convection claims, but they all assert that magnetic field generation is by simple convection by treating it as an approximation or nonlinear process. This paper addresses a truly simple axisymmetric poloidal convection and magnetic field. To calculate the electrical components, this paper introduces a theory that separates the vector potential into inductance and current in a relational formula. The reason is to consider the mutual influence between distant circuits in terms of mutual inductance. Then, the change in current is calculated from the change in inductance. It is solved as an eigenvalue problem. By this method, it is proven that a simple axisymmetric poloidal magnetic field can be generated from axisymmetric poloidal convection. Another reason is the nature of mutual inductance and Lenz's law, which reflects the effect of current changes on other electric circuits. This is intended to eliminate the concern that magnetic field growth will be impaired by magnetic field freezing. These are novel concepts, and we believe that these findings will contribute to further elucidating the formation mechanism of celestial magnetic fields and plasma reactor research. However, regarding the stability of the magnetic field, the behavior of convection itself must be considered, but since this topic is not included in this paper, only the concept is described.

Keywords: poloidal flow, dynamo theory, inductance, Lenz's law, magnetohydrodynamics

1. Introduction

Magnetic Field Study of Celestial Bodies

Researchers have long struggled to understand the mechanism underlying the formation of celestial magnetic fields. Research in this field could progress through the discovery of a new underlying mechanism. Currently, the generation of axisymmetric and poloidal magnetic fields can be solved by complex convection arguments.

For example, the famous foundations for elucidating the mechanism of the formation of celestial magnetic fields are the ω effect[1], the α effect[2], and Cowling's theorem[3].

Taking the Sun as an example, the magnetic field in the plane perpendicular to the axis of rotation of the Sun is called the toroidal magnetic field, and the magnetic field in the plane parallel to the axis of rotation is called the poloidal magnetic field. The same is true for convection. According to Cowling's theorem, axisymmetric convection does not generate a stable axisymmetric magnetic field, either poloidal or toroidal.

The ω effect generates a toroidal magnetic field from a poloidal magnetic field where there is a gradient in angular velocity. Since the rotation of the surface of the Sun is faster at the equator than at the poles, there is an angular velocity gradient. If there is a poloidal magnetic field as the initial magnetic field, the magnetic field is stretched so that it is wound up by the angular velocity gradient, and the poloidal magnetic field becomes a toroidal magnetic field. If the toroidal magnetic field is changed to a poloidal magnetic field, the magnetic field may be amplified. However, no such effect was found. In the end, the result was in favor of Cowling's theorem.

The α effect assumes a velocity field that twists a magnetic field. The concept is to twist the toroidal magnetic field in some places and direct it in the poloidal direction. Therefore, if an α effect is added to the ω effect, mutual exchange of magnetic fields is possible, and the magnetic field may be amplified. However, this approach is not as easy to use as described above. Researchers have combined these effects with complex convection to further elucidate the mechanism of magnetic field generation[4][5]. To our knowledge, few papers[6][7][8] have argued for the generation of magnetic fields by simple convection. However, these are not purely simple axisymmetric poloidal arguments (see 5. Conclusion).

The notion that a magnetic field is generated by complex convection or that an axisymmetric magnetic field does not occur constrains the study. Of course, a discussion of the generation of magnetic fields by complex convection is meaningful and necessary. However, in the observations, the difference between the axis of rotation and the magnetic axis is not large for the main celestial bodies in the solar system, especially for Saturn[9]. If there is a theory that convection and magnetic fields are simply axisymmetric, a discussion could be facilitated.

If it is clarified that a magnetic field can be generated by simpler convection, research in this field will further advance. This paper explores the possibility of generating a magnetic field by convection, which is simpler.

In this paper, we prove the generation of a purely simple axisymmetric magnetic field by the following method.

Methods for this Research

First, a formula is needed. We derive the basic electromagnetic induction equation to determine whether power generation starts and lasts. The relevant electromagnetic induction equations are expressed by the vector potential[10]. Furthermore, this vector potential is converted into an expression of inductance[10]. The purpose of this is to separate the vector potential into inductance, which is the component that represents the structure of the fluid, and current, which is the electrical component. The reason is to consider the mutual influence between distant circuits in terms of mutual inductance. Then, the change in current is calculated from the change in inductance. Another reason is to consider mutual inductance in combination with Lenz's[10] law. This is because this combination eliminates the concern that magnetic field growth is impaired by magnetic field freezing[11]. For this idea, see 4. Discussion for details. This electromagnetic induction equation is set and combined in a plurality of circuits. This problem is solved by numerical calculation as an eigenvalue problem. As a solution, the eigenvalues and eigenvectors are obtained. These results imply a change in the current and current distribution in the circuits. The results are shown in tables and figures, showing the possibility of generating magnetic fields and the distribution of axisymmetric magnetic fields. As a result, it is possible to show the generation of an axisymmetric magnetic field from axisymmetric convection without particularly complex convection.

Here, we explain the meaning of multiple circuits and inductances. The inductance is important for understanding this stady. Here, an overview of how inductance is handled is provided. Poloidal convection is considered axisymmetric convection (Fig. 1). This figure shows convection in which a torus-shaped (doughnut-shaped) fluid moves in the direction indicated by $\boldsymbol{U}$. Since it is a convection of a conductive fluid, it is thought that multiple coaxially toroidal circular electrical circuits (hereinafter referred to as coils) move as bundles in a poloidal manner. The inductance referred to above is the inductance of these coils. This term refers to both the self-inductance and mutual inductance between the coils. There are an infinite number of these coils, but in the numerical calculations described below, 16 circuits are set to move in the direction indicated by $\boldsymbol{U}$ on the torus surface, as shown in the cross-sectional view of the circuit (Fig. 2). The convection and coil settings are described in detail in section 2.1.

The Role of the Formulas

When a coil moves in a poloidal manner with convection, the coil moves in the radial direction and the cylindrical axial direction of the cylindrical coordinates. In this way, the coil moves in the existing magnetic field, and power generation occurs. The derived equation shows the relationship between this convection and power generation. However, a magnetic field is not always present. A magnetic field in the appropriate direction must also be generated by power generation. This process is called self-excitation power generation. We would like to prove that it is possible to generate electricity by integrating the entire torus. For this purpose, the equation is applied to a plurality of coils with different positions and solved as an eigenvalue problem. If power generation is recognized as a result of the calculation, the possibility of self-excited power generation as a whole of the torus can be explained.

Thus, as a result of the numerical calculations, the eigenvalues and eigenvectors are obtained. Since the eigenvalue of the positive polarity indicates the growth of the eigenvector (the current distribution of the coil), this proves the growth of the magnetic field. Furthermore, the magnetic field distribution calculated by calculating the obtained eigenvector components as the current flowing through each coil is also shown in the figure. Based on these results, the possibility of self-excited power generation that creates an axisymmetric poloidal magnetic field is explained.

Stable Magnetic Field

Here, the possibility of growth of the magnetic field is shown, but the stability of the magnetic field is not indicated. We believe that a stable magnetic field is possible in relation to convection. However, since convection behavior is not the subject of this paper, we will only discuss the possibility of maintaining the stability of the magnetic field. Even if this is insufficient, we believe that these results and ideas will be useful for future related research. We also believe that this argument can be applied to more than celestial bodies. This is shown in 5. Conclusion.

The Configurations

Here, the following descriptive structure is briefly shown.

2. Mechanism

2.1 Description of the problem

Description of the geometric structure of the object of consideration and its figures

2.2 Calculation of the magnetic properties

Derivation of mathematical formulas for numerical calculations

Explanation of the inductance calculation method

Other elements of the relevant formulas

3. Calculation Conditions and Results

3.1 Calculation conditions

2. Mechanism

2.1. Description of the problem In this paper, we solve and discuss axisymmetric convection and magnetic fields by numerical calculations. Here, the geometric structure of convection and the knowledge necessary to calculate coil inductances are explained using figures.

To determine whether a magnetic field can fluctuate in the poloidal stream of a conductive fluid, a certain poloidal flow is set, and the induction equation (as a simultaneous equation expressed by inductances) is expressed in terms of toroidal vector potentials to calculate the current as an eigenvalue problem. The poloidal flow of a fluid occurs in a torus shape (Fig. 1). The upper figure is the whole, and the bottom figure is the cross section, where $\boldsymbol{U}$ is the poloidal velocity, R_0 is the radius of the poloidal flow, and r is the radius of an example position on the torus from the Z axis. There are an infinite number of coils in the coil bundle. Here, only a part of the coils shown below will be considered.

A representative cross section (Y–Z plane) of the torus is shown (Fig. 2(a)). The stream is divided into toroidal segments for calculation as coils (Fig. 2(b)). That is, Fig. 2(a) and (b) show the right half of the cross section of Fig. 1. Z is the center axis, and Ra is the radial axis of the cylindrical coordinates (equivalent to the Y axis in Fig. 1), where the circle indicates the cross section of the torus. P_c is the center of the flow, where r_0 and z_0 are the elements of position P_c in the Ra and Z directions, respectively. Notably, P_0 is at the coordinates (0,0), and P_c is at (r_0,0). P is a representative position at which the flow velocity vector $\boldsymbol{U}$ is calculated; u_r and u_z are the elements of $\boldsymbol{U}$ in the Ra and Z directions, respectively; θ is the angle between the Ra axis and position P; and r is the element of position P in the Ra direction (Fig. 2(a)). Note that θ is not the zenith angle of polar coordinates but the angle from the Ra axis. The coils used to define the flow torus are defined in Fig. 2(b). Sixteen coils are considered, where n refers to the coil number. The dotted lines indicate the coaxial coils (i.e., the region occupied by the fluid), which are separated by the thickness T. There are multiple coils that wind only once around the Z axis, and the coils move in the direction of $\boldsymbol{U}$ with radius R_0. The electric current runs separately in each coil in the ϕ direction that orbits the Z axis. Although the coils can move, the later calculation of the eigenvalues assumes the state of the coils in a brief moment, Δt.

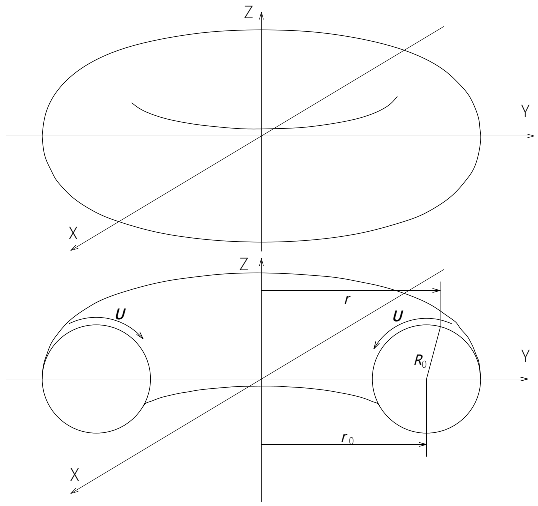

Figure 1: Schematic of the toroidal geometry. It is an approximate geometric outline of convection set by numerical calculations. A torus-like conductive fluid flows in the U direction. The upper figure is the whole, and the bottom figure is the cross section.

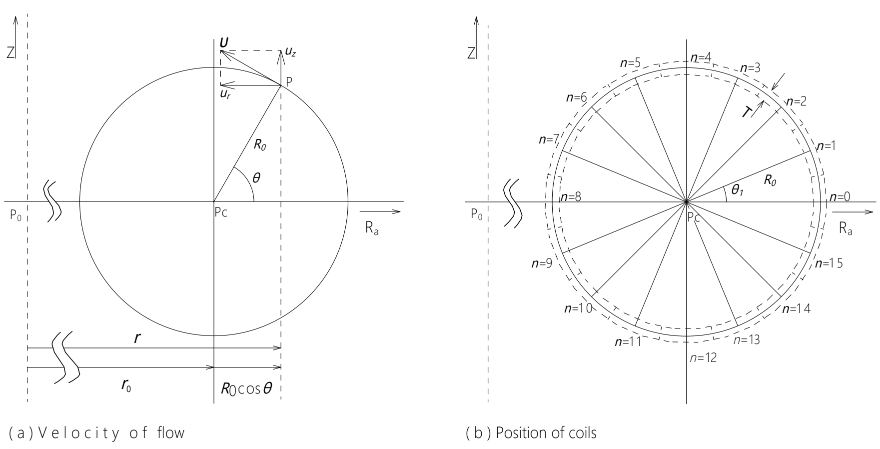

Figure 2: Schematics of the (a) velocity of the fluid flow and (b) defined coils in the cross section of the convection (Y–Z plane). It is a cross section of Fig. 1 and shows settings such as the arrangement of virtual coils used in numerical calculations.

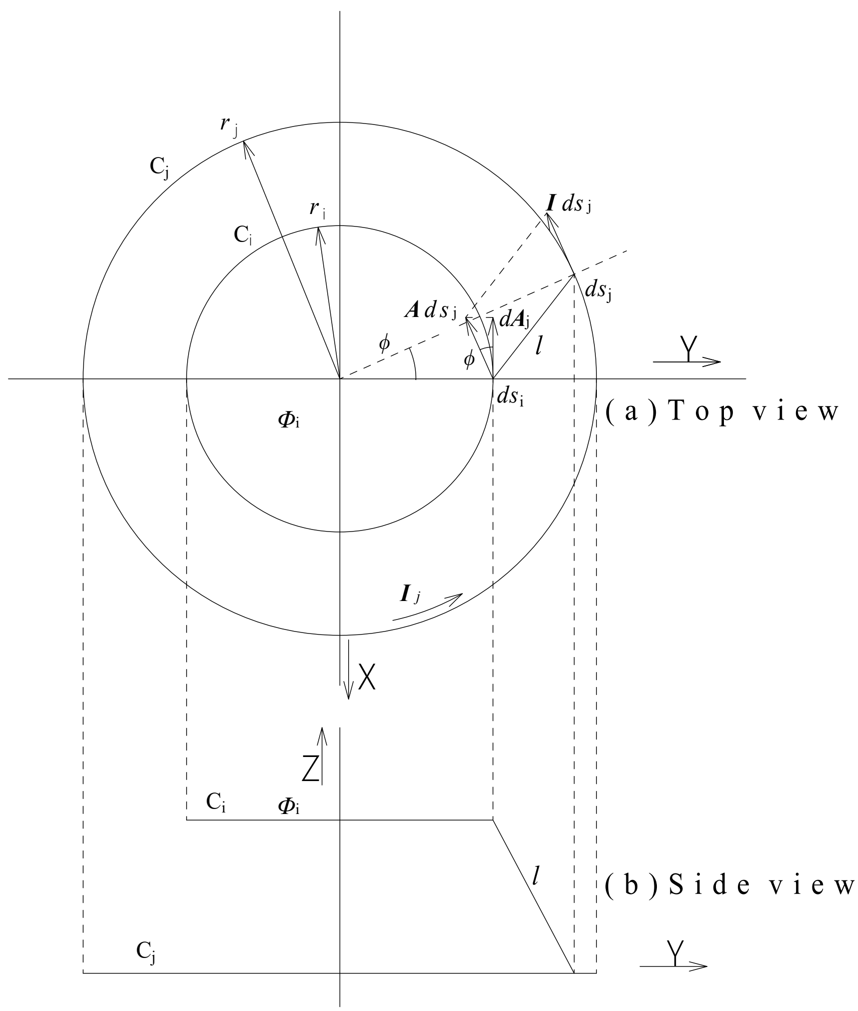

Figure 3: Relationship between the electric current I and vector potential $\boldsymbol{A}$ for a set of any two coils shown in Fig. 2: (a) top view and (b) side view of the torus. Two virtual coils are taken up to explain how to calculate self and mutual inductance.

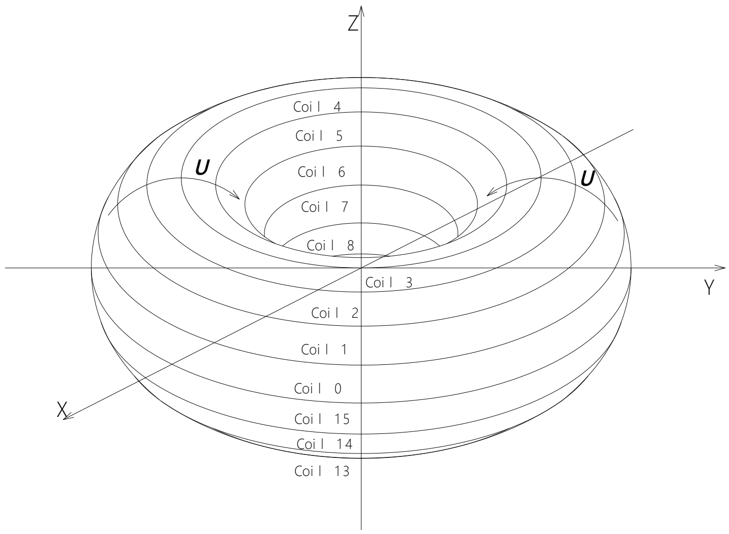

Figure 4: 3D view of the coaxial coils. It is expressed in a bird's eye view to make it easier to imagine the arrangement of the virtual coil.

The top and side views of a set of any two coaxial coils are shown (Figs. 3(a) and (b)). They explain the relationship between the electric current (I) and vector potential[10] $\boldsymbol{A}$ for the calculation of inductances[10]. X, Y, and Z are the axes of the rectangular Cartesian coordinate system (Fig. 3). Cj is Coil j, in which the current I_j flows, and Ci is Coil i, which obtains the vector potential[10] $\boldsymbol{A}j$ induced by the current I_j running in Coil Cj. ϕ is the angle of rotation around the Z axis, starting from the Y axis. Here, r_i and r_j are the radii of Ci and Cj, respectively. Furthermore, dsi and dsj are minute lengths of Ci and Cj for integration, respectively, where dsi is placed on the X-axis ($\phi = 0$) and dsj is placed at ϕ with a distance l between them. $Idsj$ is the contribution to the current over dsj, where $\boldsymbol{A}dsj$ is the vector potential induced by $Idsj$. $d\boldsymbol{A}j$ is the element of $\boldsymbol{A}dsj$ in the X direction, and it is integrated with respect to ϕ to obtain $\boldsymbol{A}j$. Φ_i is the total flux linking Ci. In addition, a 3D image of the coils is shown to clarify their relationships with each other (Fig. 4). Each coil is arrayed coaxially with the Z axis, and the coils are parallel to each other.

2.2. Calculation of the magnetic properties In the numerical calculations in this paper, the electromagnetic induction equations are applied to multiple coils, and the current is calculated by solving them as an eigenvalue problem by combining them. Here, the underlying electromagnetic induction equation is derived. In addition, a method for calculating the inductance to be included in the numerical calculation is described.

Derivation of Basic Formulas

As a source equation for determining the relationship between the electric current and magnetic field, the current is calculated using Ohm's law[10] with an electric field as follows:

$$\boldsymbol{J} = \sigma \left(\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B} \right) \tag{1}$$

Here, $\boldsymbol{J}$ is the current density; σ is the electrical conductivity; $\boldsymbol{u}$ is the velocity of the conductive fluid; $\boldsymbol{B}$ is the magnetic field, which can be found using Eq. (2);

$\boldsymbol{u} \times \boldsymbol{B}$ is the motion of the electric field; and $\boldsymbol{E}$ is the electric field potential, which can be found using Eq. (3)[10] as follows:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} \tag{2}$$

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} + \boldsymbol{\nabla}\varphi \tag{3}$$

Here, φ is a scalar potential, $\boldsymbol{A}$ is a vector potential[10], and t represents the time.

To derive an induction equation expressed in vector potentials from Ohm's law, Eq. (1), these equations are combined as follows:

$$\frac{\partial \boldsymbol{A}}{\partial t} - \boldsymbol{u} \times (\boldsymbol{\nabla} \times \boldsymbol{A}) = \boldsymbol{\nabla}\varphi - \frac{\boldsymbol{J}}{\sigma} \tag{4}$$

To apply this induction equation for coils, the factor $2\pi r_i$(r_i=coil Ci radius) is multiplied as an integral around the coil on both sides of the equation because the $\boldsymbol{A}$ and current are the same around Ci and Cj. Assuming that $\boldsymbol{J} = \boldsymbol{I}/S$, where $\boldsymbol{I}$ is the toroidal current and S is the cross-sectional area of the coil, Eq. (4) can be rewritten as follows:

$$\frac{\partial}{\partial t}(2\pi r_i \boldsymbol{A}) - \boldsymbol{u} \times (\boldsymbol{\nabla} \times 2\pi r_i \boldsymbol{A}) = -\frac{2\pi r_i}{\sigma}\frac{\boldsymbol{I}}{S} \tag{5}$$

Here, φ is ignored because it is assumed that going around along Ci at that gradient will result in a value of zero. Furthermore, Eq. (6) shows the relationship between $\boldsymbol{A}$ and the inductance[10] (L). Here, the total magnetic flux linking the coil, Φ, is used in place of L, where $\Phi = LI$. The subscripts i and j refer to the coil numbers defined in Fig. 2, enabling the development of simultaneous equations.

$$\Phi_i = \oint_{C_i} \boldsymbol{A}_j ds_i \tag{6}$$

Here, Φ_i is the flux of coil Ci, and dsi is a minute part of coil Ci. In this arrangement of coils, all the coils are arrayed coaxially with the Z axis and parallel to each other. Thus, $\boldsymbol{A}_j$ is the same around Ci. Therefore, $\Phi_i = 2\pi r_i \boldsymbol{A}_j$. When $\Phi_i = L_{ij} I_j$, the relationship between L_{ij} and $\boldsymbol{A}_j$ is as follows. L_{ij} is a matrix meaning inductance, but only the diagonal element is self-inductance, and the other elements are mutual inductance, so it is hereafter referred to as M_{ij}.

$$\boldsymbol{A}_j = \frac{I_j}{2\pi r_i} M_{ij} \tag{7}$$

As will be discussed later, both $\boldsymbol{A}_j$ and $\boldsymbol{I}_j$ have only a toroidal component along the coil. Using Eq. (7), the vector potential is separated into inductance, which is a component that expresses the structure of the fluid (electric circuits), and current, which is an electrical component. The reason is to consider the mutual influence between distant circuits in terms of mutual inductance. Then, the change in current is calculated from the change in inductance. By substituting Eq. (7) into Eq. (5), the following equation is obtained:

$$\frac{\partial}{\partial t}\left(M_{ij}I_j\right) - \boldsymbol{u} \times \left(\boldsymbol{\nabla} \times 2\pi r_i \boldsymbol{A}_j\right) = -\frac{2\pi r_i}{\sigma S}\boldsymbol{I}_j \tag{8}$$

Here, σ is the conductance of the fluid, and r_i is the radius from the Z axis. However, the second term on the left-hand side of Eq. (8) is complicated. Thus, it must be addressed separately. It is considered by decomposing the term as follows:

$$\boldsymbol{u} \times \left(\boldsymbol{\nabla} \times \boldsymbol{A}_j\right) = \begin{bmatrix} u_{ir} \\ 0 \\ u_{iz} \end{bmatrix} \times \begin{bmatrix} \frac{1}{r_i}\left(-r_i \frac{\partial \boldsymbol{A}_{j\phi}}{\partial z_i}\right) \\ 0 \\ \frac{1}{r_i}\left(\frac{\partial r_i \boldsymbol{A}_{j\phi}}{\partial r_i}\right) \end{bmatrix}$$

$$= \begin{bmatrix} u_{ir} \\ 0 \\ u_{iz} \end{bmatrix} \times \begin{bmatrix} -\frac{\partial \boldsymbol{A}_{j\phi}}{\partial z_i} \\ 0 \\ \frac{1}{r_i}\boldsymbol{A}_{j\phi} + \frac{\partial \boldsymbol{A}_{j\phi}}{\partial r_i} \end{bmatrix} = \begin{bmatrix} 0 \\ -u_{iz}\frac{\partial \boldsymbol{A}_{j\phi}}{\partial z_i} - u_{ir}\left(\frac{1}{r_i}\boldsymbol{A}_{j\phi} + \frac{\partial \boldsymbol{A}_{j\phi}}{\partial r_i}\right) \\ 0 \end{bmatrix} \tag{9}$$

Here, rot $\boldsymbol{A}_j$ is decomposed into cylindrical coordinates, and subscripts r, ϕ, and z indicate the component directions in cylindrical coordinates, namely, the radius from the Z axis, the angle around the Z axis, and the Z direction, respectively. Since the current only runs through the toroidal coil, only the toroidal $\boldsymbol{A}_{j\phi}$ component remains with the vector potential. Furthermore, since it is uniform in the toroidal direction, the $\frac{\partial}{\partial \phi}$ component is zero, and the description is excluded. Therefore, $\boldsymbol{A}_j$ has a component that is only in the ϕ direction. Thus, the other components of $\boldsymbol{A}_j$ are omitted. Eq. (10) is obtained by multiplying Eq. (9) by $2\pi r_i$, substituting Eq. (7) and including only the ϕ component as follows:

$$\begin{aligned} &2\pi r_i \left[-u_{iz}\frac{\partial \boldsymbol{A}_{j\phi}}{\partial z_i} - u_{ir}\left(\frac{1}{r_i}\boldsymbol{A}_{j\phi} + \frac{\partial \boldsymbol{A}_{j\phi}}{\partial r_i}\right)\right] \\ &= -u_{iz}\frac{\partial M_{ij}I_{j\phi}}{\partial z_i} - u_{ir}\left(\frac{1}{r_i}M_{ij}I_{j\phi} + \frac{\partial M_{ij}I_{j\phi}}{\partial r_i}\right) \\ &= -u_{ir}\frac{\partial M_{ij}I_{j\phi}}{\partial r_i} - u_{iz}\frac{\partial M_{ij}I_{j\phi}}{\partial z_i} - u_{ir}\frac{1}{r_i}M_{ij}I_{j\phi} \end{aligned} \tag{10}$$

These terms are replaced by the second term on the left-hand side of Eq. (8) to obtain Eqs. (11) and (12) as follows:

$$\begin{aligned} &\frac{\partial}{\partial t}\left(M_{ij}I_{j\phi}\right) - \left(-u_{ir}\frac{\partial M_{ij}I_{j\phi}}{\partial r_i} - u_{iz}\frac{\partial M_{ij}I_{j\phi}}{\partial z_i} - u_{ir}\frac{1}{r_i}M_{ij}I_{j\phi}\right) \\ &= -\frac{2\pi r_i}{\sigma S}I_{j\phi} \end{aligned} \tag{11}$$

$$\frac{\partial}{\partial t}\left(M_{ij}I_{j\phi}\right) + u_{ir}\frac{\partial M_{ij}I_{j\phi}}{\partial r_i} + u_{iz}\frac{\partial M_{ij}I_{j\phi}}{\partial z_i} + u_{ir}\frac{1}{r_i}M_{ij}I_{j\phi} = -\frac{2\pi r_i}{\sigma S}I_{j\phi} \tag{12}$$

The first thing we want to confirm with the derived equation is the start of power generation. There, the current is almost zero. Since the value of the fourth term on the left side of Eq. (12) is set to a very small initial stage, it is ignored. However, this term will be described later (4. Discussion: Sub-self-excitation power generation) since it plays an important role to some extent if the current increases. Since the first three terms on the left-hand side of Eq. (12) are equivalent to a total differential, they can be replaced as follows:

$$\frac{d}{dt}\left(M_{ij}I_{j\phi}\right) = -\frac{2\pi r_i}{\sigma S}I_{j\phi}$$

When the equation is transformed and rearranged, Eq. (13) is obtained.

$$\frac{dM_{ij}}{dt}I_{j\phi} + M_{ij}\frac{dI_{j\phi}}{dt} = -\frac{2\pi r_i}{\sigma S}I_{j\phi}$$

$$\frac{dI_{j\phi}}{dt} = M_{ij}^{-1}\left(-\frac{dM_{ij}}{dt}I_{j\phi} - \frac{2\pi r_i}{\sigma S}I_{j\phi}\right)$$

$$\Lambda I_{j\phi} = -M_{ij}^{-1}\frac{dM_{ij}}{dt}I_{j\phi} - M_{ij}^{-1}R_{ij}I_{j\phi} \tag{13}$$

Here, Λ indicates the eigenvalues. These simultaneous equations are used here to obtain the Λ values. R_{ij} is a matrix of the resistance and a function of the coil circumference and cross-sectional area S, where S is calculated from the thickness T and the section of the flow course as $S = (2\pi R_0 T)/16$. Then, $R_{ij} = 2\pi r_i/(\sigma S) = (16 r_i)/(\sigma R_0 T)$. R_{ij} is a diagonal matrix because the voltage drop exists only for $i = j$.

Method for Calculating the Inductance

The inductance used in the example calculation is as follows. In the coil description (Fig. 2), the coil's cross-sectional area and shape are disregarded in the calculation of the inductance because these geometrical factors introduce a large degree of complexity. Therefore, the coil is treated as a thin line as an approximation (Figs. 3(a) and (b)). $\boldsymbol{A}_j$ is calculated as follows:

$$\begin{aligned}\boldsymbol{A}_j &= \frac{\mu}{4\pi}\oint_{C_j}\frac{\boldsymbol{J}}{l}dV = \frac{\mu}{4\pi}\oint_{C_j}\frac{\boldsymbol{J}S}{l}ds_j = \frac{\mu}{4\pi}\oint_{C_j}\frac{\boldsymbol{I}_j}{l}\cos\phi ds_j \\ &= \frac{\mu}{4\pi}I_j\oint_{C_j}\frac{\cos\phi}{l}ds_j,\end{aligned} \tag{14}$$

where l is the distance between dsi and dsj, μ is the magnetic permeability, dV is the minute volume, S is the cross-sectional area of the current (i.e., the coil), $\boldsymbol{I}_j = \boldsymbol{J}S$ is the current, and the directional element for dsi of $\boldsymbol{I}_j$ is $\boldsymbol{I}_j cos\phi$. By substituting Eq. (7) for $\boldsymbol{A}_j$ in Eq. (14), Eq. (15) is obtained as follows:

$$M_{ij} = \frac{2\pi r_i}{I_j}\frac{\mu}{4\pi}I_j\oint_{C_j}\frac{\cos\phi}{l}ds_j = 2\pi r_i\frac{\mu}{4\pi}\oint_{C_j}\frac{\cos\phi}{l}ds_j \tag{15}$$

When μ is set to $4\pi \times 10^{-7}$H/m (vacuum conditions), Eq. (16) is obtained as:

$$M_{ij} = 2\pi r_i \oint_{C_j} \frac{\cos\phi}{l} ds_j \times 10^{-7} \tag{16}$$

M_{ij} is calculated by summing Eq. (17), where C_j is divided into $k = 100$ equal parts, Δsj, as follows:

$$M_{ij} \approx 2\pi r_i \sum_{k=1}^{100} \frac{\cos\phi_k \Delta s_j}{l_k} \times 10^{-7} \tag{17}$$

$$l_k = \sqrt{(r_i \cos\phi_k - r_j)^2 + (r_i \sin\phi_k)^2 + (z_i - z_j)^2} \tag{18}$$

The angle of a specific coil n is given by $\theta_n = 2\pi n/16$, where $r_n = r_0 + R_0 \cos\theta_n$ and $z_n = R_0 \sin\theta_n$ (Fig. 2). Previously, i and j were used in mutual inductance calculations to distinguish between the coils that received electromotive force and the coils that had a current flow. Each number n is replaced by i or j for any two of the coils.

Here, Eqs. (13) and (15) are used to explain the tendency of the inductance to change due to convection. The variable r_i is included in Eq. (15), and z_i is contained in l_k (Eq. (18)). Thus, power generation is related to the velocity of convection in the Ra and Z directions. That is, $\frac{dM_{ij}}{dt}$ is driven by poloidal convection. In this drive, not only the change in self-inductance but also the change in mutual inductance is important because the mutual coupling tends to fluctuate due to the difference in convection when the coils are far apart.

Elements of Other Expressions

$\frac{dM_{ij}}{dt}$ is calculated as the difference in M_{ij} induced by the flow velocity over a period of a minute divided by $\Delta t(ex.1.0 \times 10^{-6}s)$. The velocity $\boldsymbol{U}$ depends on the θ_n of each coil and is calculated as follows (Fig. 2):

$$|\boldsymbol{U}| = \frac{r_0}{r_i}\omega R_0,\ u_r = -|\boldsymbol{U}|\sin\theta_n,\ u_z = |\boldsymbol{U}|\cos\theta_n. \tag{19}$$

where ω is the angular velocity of the flow. $\frac{r_0}{r_i}$ is a coefficient that adjusts based on the position of r_0 so that the flow rate satisfies the continuity equation. Furthermore, M_{ij}^{-1} is the inverse matrix of M_{ij}.

3. Calculation Conditions and Results

3.1. Calculation conditions The conditions for the numerical calculations were as follows.

Calculation Conditions

The general conditions were $R_0 = 1000$ m, $r_0 = 2000$ m, and $T = 0.1R_0$. The electrical conductivity σ was 10^3 s/m (Solar convection zone), which was obtained from a textbook[9]. The velocity of convection is given by Eq. (19),$|\boldsymbol{U}| = \frac{r_0}{r_i}\omega R_0$. As a condition, the angular velocity ω was set to ω_m, where only the maximum eigenvalue was barely positive. In other words, we set the conditions that seem to be the boundary at which power generation begins to occur.

Similarity of Scale

Table 1: Eigenvalues Λ. The eigenvalue is calculated by numerical calculation. Assuming the boundary at the start of magnetic field generation, we set the calculation condition to generate one barely positive eigenvalue. Therefore, only $\lambda 16$ is positive.

Λ	Eigenvalues
$\lambda 1$	-5.291×10^{-1}
$\lambda 2$	-3.224×10^{-1}
$\lambda 3$	-2.293×10^{-1}
$\lambda 4$	-1.767×10^{-1}
$\lambda 5$	-1.433×10^{-1}
$\lambda 6$	-1.205×10^{-1}
$\lambda 7$	-1.073×10^{-1}
$\lambda 8$	-9.003×10^{-2}
$\lambda 9$	-8.899×10^{-2}
$\lambda 10$	-6.486×10^{-2}
$\lambda 11$	-6.463×10^{-2}
$\lambda 12$	-4.583×10^{-2}
$\lambda 13$	-4.142×10^{-2}
$\lambda 14$	-2.967×10^{-2}
$\lambda 15$	-1.854×10^{-2}
$\lambda 16$	$\mathbf{3.578 \times 10^{-4}}$

As estimated under other conditions, as long as the numerical values of R_0, r_0, and T change in a similar form, the eigenvalues also change in a similar form. In other words, since there is no change in the interval ratio or polarity of the eigenvalues, the nature of magnetic field generation is considered the same even if the scale of convection changes. Therefore, we did not present the calculations for other dimensions in this paper.

3.2. Results The result calculated based on Eq. (13) is shown under the conditions shown above. The eigenvalues $\Lambda(\lambda_1 - \lambda_{16})$ are listed in Table 1. A positive value is highlighted in bold in the table. Only the maximum eigenvalue, λ_{16}, has positive polarity. Although it has positive polarity, the absolute value is small. This is because the calculation conditions were set in a specific way. This means that under these conditions, power generation begins. In addition, if the convection speed is higher, the power generation will be stronger. The eigenvector of the maximum eigenvalue is shown in Table 2. The length of each component of this eigenvector is adjusted so that the norm is 1. This condition indicates that the absolute value of the current is maximized in Coil 6, which is highlighted in bold. These components indicate the current values of each coil. This table is used to create the following figure. The lattice-like distribution of the magnetic field generated from the current state (based on the eigenvector) is shown in Fig. 5. It is vectorially displayed on the Z-Ra plane, which is the same surface (Fig. 2). These lengths are compressed by square roots. The circle centered on the P_c indicates the convection path. Each small round mark on the convection circle indicates the location of the coil (Fig. 5). It was calculated as if an electric current was passing through this position. Since this surface is rotationally symmetric on the Z axis, it shows a rotationally symmetrical magnetic field distribution in the poloidal pattern.

Table 2: Eigenvector of the maximum eigenvalue in Table 1. This corresponds to the current distribution flowing through each coil. The combined current is adjusted to 1. The current is maximized at coil 6.

Coil numbers	Eigenvector
0	7.963×10^{-2}
1	1.164×10^{-1}
2	1.675×10^{-1}
3	2.405×10^{-1}
4	3.417×10^{-1}
5	4.627×10^{-1}
6	$\mathbf{5.447 \times 10^{-1}}$
7	4.645×10^{-1}
8	2.080×10^{-1}
9	5.420×10^{-3}
10	-4.865×10^{-2}
11	-3.871×10^{-2}
12	-1.662×10^{-2}
13	5.754×10^{-3}
14	2.779×10^{-2}
15	5.144×10^{-2}

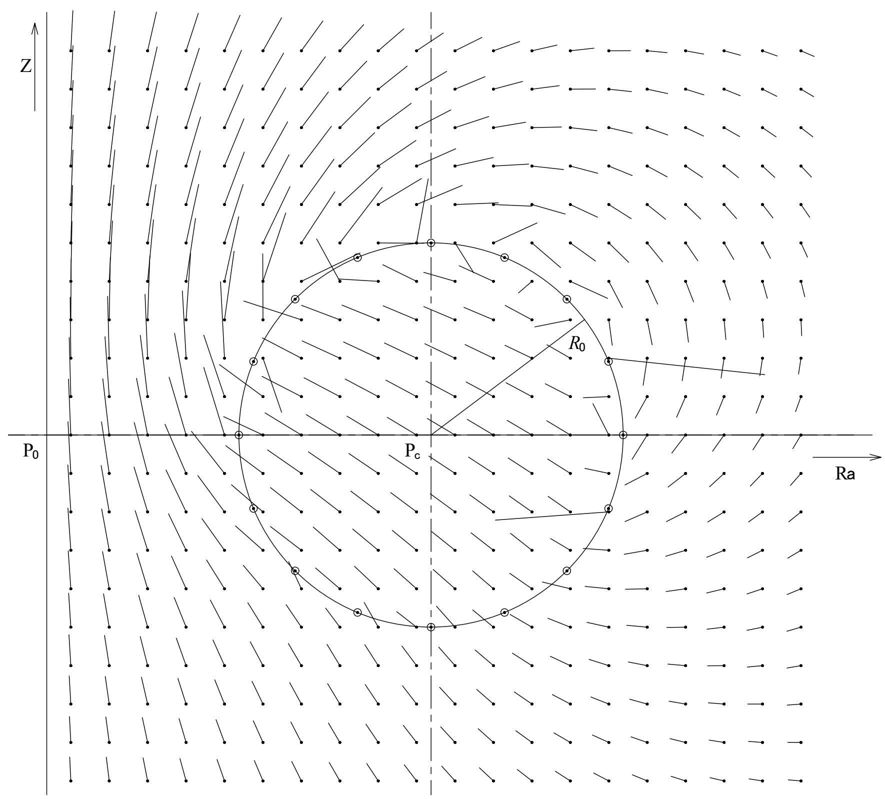

Figure 5: Magnetic field distribution generated from current state based on eigenvectors (Table 2). This is the magnetic field distribution drawn on the Z-Ra plane based on the results of numerical calculations. Since this surface is rotationally symmetric on the Z axis, it shows a rotationally symmetrical magnetic field distribution in the poloidal flow.

4. Discussion As described above, the underlying electromagnetic induction equation was derived, and the result was obtained by numerical calculation as a system eigenvalue problem. As a result, power generation starts, and power generation that generates an axisymmetric magnetic field in poloidal is possible. Here, we will consider the effects that can be expected from this result. In addition, sub-self-excitation power generation and problems are described. Moreover, we describe our thoughts on the possibility of magnetic field stability.

Generation of Axisymmetric Magnetic Fields

A positive eigenvalue means that the eigenvector increases exponentially as an exponential value of the eigenvalue. The eigenvector of the maximum eigenvalue of positive polarity is shown in Table 2. Each component indicates the current in each coil. However, this eigenvalue or eigenvector only means that power generation starts, and the state starts at the beginning. The electric current increases over time. The larger the eigenvector is, the faster the magnetic field grows. If self-excitation power generation occurs and the magnetic field grows, the magnetic field distribution is expected, as shown in Fig. 5. The following discussion can be drawn from the figure.

Strong magnetic fields parallel to the Z axis are generated near the axis. The weak magnetic fields circle this convection far from it. They also pass into the circle of convection. In places where the magnetic field is strong, the convection and magnetic fields approximately intersect. Note that some vectors near small round marks on convection are unnatural in size and direction. This is because the lattice points for calculating the magnetic field distribution and the position of the coil were too close together, so the extreme values were calculated. The Z-Ra plane circles around the axis of symmetry in the same pattern (Fig. 5.). In other words, it is an axisymmetric magnetic field. Therefore, it was shown that an axisymmetric poloidal magnetic field grows due to axisymmetric convection.

Sub-self-excitation power generation

When power generation starts and the current increases, the term deferred explanation is also somewhat important for the generation of the magnetic field. With respect to time, the fourth term on the left side of Eq. (12), $+u_{ir}\frac{1}{r_i}M_{ij}I_{j\phi}$, plays an important role to some extent. In this term, the velocity, u_{ir}, and the current, $I_{j\phi}$, are multiplied. When power generation begins, the current increases, and this term can dominate. Since power generation becomes increasingly powerful due to the current generated by power generation, this term can be seen as self-excited power generation. This term arose from the fact that it is a cylindrical coordinate if it is traced back to the original. In other words, it is a term for self-excited power generation arising from axisymmetry. However, since $\frac{1}{r_i}$ is multiplied in the term, the effect is considered weak at a position far from the axis of symmetry. It is not known how effective this term is in natural convection.

Even if there is no sub-self-excitation power generation and if the absolute value of the maximum eigenvalue of the positive polarity is large, the magnetic field increases exponentially because the exponent is large. The main self-excitation power generation is performed. In this case, the magnetic field can be shown (Fig. 6). It is generated by the convection of conductive fluid in the celestial body. Note that this figure is schematically drawn.

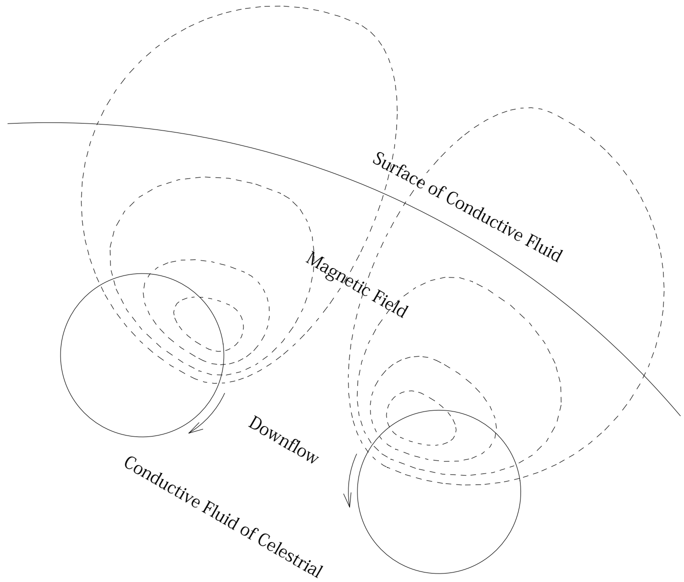

Figure 6: Examples of poloidal axisymmetric convection and magnetic field in a celestial body. The solid arrows indicate the fluid flow, while the dashed lines indicate the induced magnetic field. Based on Fig. 5, this figure is schematically drawn.

Possibility of Magnetic Field Stabilization

The self-excited power generation discussed above becomes more powerful because the current increases when power generation starts. Then, even if convection weakens from the start, power generation will continue. Then, it continues even if the power generation consumes the energy of convection and the convection speed decreases. On the other hand, if the speed decreases, the power generation decreases. Eventually, there will be a balance somewhere depending on the ability to supply convective energy from the external environment. In other words, there is a possibility that it will be stabilized at the strength of a certain magnetic field. It is necessary to examine the behavior of convection and the energy supply capacity of convection to determine whether it is actually stable, but this is not the subject of this paper.

The Problem of the Magnetic Field being carried away by Magnetic Field Freezing

Thus far, we have investigated the possibility of magnetic field growth via numerical calculations, but there is a question here. This is a problem of magnetic field freezing[11]. This is a phenomenon in which the magnetic field cannot move or becomes difficult to move in a high electrical conductor. Since the diffusion rate of the magnetic field is slow, the magnetic field is carried away with the movement of the coil, so it seems that continuous growth of the magnetic field at a particular location cannot be expected. In the setting of this paper, it is considered that the magnetic field entangled with the current of each coil moves with convection. Even if the coil reaches the highest point of power generation, it will pass by immediately. In this case, there is a concern that there is little time for the magnetic field to grow continuously.

This phenomenon may be easier to understand from the point of view of the formula. The equations were set for each coil and were in a system. Since the calculated eigenvalue represents the current of the coil, it is a value that moves with the coil. However, this is because the fluid was divided into individual coils in this paper, and the phenomenon of magnetic field freezing did not appear in the equation. Even so, this is easy to understand if a similar phenomenon is considered.

However, in this paper, we assume that the magnetic field grows in a specific location. The basis is the inductance considered in this paper. Eq. (13) is used for the numerical calculations and includes the mutual inductance. If we consider the change in time here, that is, the change in the current and the resulting change in magnetic flux, then Lenz's law[10] can be considered.

Here, we explain the meaning of the term "combination of mutual inductance and Lenz's law", which we use as the key point of this paper. The inductance is the relationship between the magnetic field and current ($\Phi_1 = L_1 I_1 + M_{12} I_2 + M_{13} I_3 + \ldots$). Lenz's law refers to the effect of generating an electromotive force E in the direction of suppressing the change in the magnetic field in the circuit ($E = -\frac{d\Phi}{dt}$). When combined, these results indicate that an electromotive force that suppresses the change in current is generated, including other circuits ($E_1 = -L_1 \frac{dI_1}{dt} - M_{12} \frac{dI_2}{dt} - M_{13} \frac{dI_3}{dt} \ldots$). This process is called "reciprocal induction". This electromotive force causes a current in the circuit in the direction of suppressing the change in the current of these circuits. In this paper, the action of the mutual inductance part is used as the basis for the discussion.

For example, if the current increases in the coil at one position, a current in the opposite direction is generated in the coil at the other position. Conversely, if the current decreases in the coil at one position, a current in the same direction is generated in the coil at the other position. Therefore, when the current in the coil increases once it departs from the peak position of the current, a current in the same direction is generated in the coil at the other position to suppress the decrease in the current. At the position where the current is at its peak, the coil exiting that position and the coil entering the coil are adjacent to each other, so the current of the incoming coil increases as the current of the outgoing coil decreases. That is, at that position, the current is inherited by the next coil. This is a new concept that has not been introduced in other theories. Hereinafter, this effect is abbreviated as the inheritance effect.

Therefore, current growth (magnetic field growth) at a specific position is considered possible. The matrix on the right side of Eq. (13) for calculating the eigenvalues in this paper includes the mutual inductance (other than M_{ii}.) Then, this matrix is solved as an eigenvalue problem to obtain the change (eigenvalues) in the current (eigenvectors). Therefore, it can be expected that this eigenvalue has the property of not moving at a specific position. In other words, the magnetic field continues to grow at a specific position according to the calculated eigenvalue. Furthermore, in real convection, since countless coils are close to each other, the coupling of mutual inductance is considered to be strong. Therefore, the inheritance effect is considered to work efficiently.

If we do not consider the inheritance effect, we need to discuss the growth of the magnetic field through another effect. For example, complex convection, such as the combination of the effects of α and ω, must be considered. In summary, even if there is a magnetic field freeze, the calculated eigenvalues and eigenvectors do not move with the movement of each coil but represent the behavior of the current remaining at the coil set position (Fig. 2).

Therefore, it has been shown that an axisymmetric poloidal magnetic field grows due to axisymmetric convection. In addition, the magnetic field stability can be maintained depending on the conditions.

5. Conclusion This paper argues that the growth of an axisymmetric magnetic field occurs with purely simple convection. Unlike previous theories, we have proven that it is possible to grow an axisymmetric magnetic field from axisymmetric poloidal convection without including turbulence. We believe that this is a novel proposal, but there are seemingly contradictory and similar points to other theories. There are also limitations in terms of application. On the other hand, there are things to look forward to. These will be described below.

Relationship with the Anti-dynamo Theorem

This is the relationship with the anti-dynamo theorem. Below are our thoughts on these theorems.

Theorem 1. In Cartesian coordinates (x, y, z), no field independent of z that vanishes at infinity can be maintained by dynamo action. Therefore, it is impossible to generate a 2D dynamo field.

In this paper, cylindrical coordinates are used for discussion, but this theorem is an argument in Cartesian coordinates. Even if the setting of this thesis is changed to Cartesian coordinates, it is not subject to this theorem because it does not find a 2D plane with a z-direction that applies to this theorem.

Theorem 2. No dynamo can be maintained by a planar flow ($u_x(x, y, z, t)$, $u_y(x, y, z, t)$,0)[12]. No restriction is placed on whether the field is 2D or not in this theorem.

In this paper, convection and the magnetic field act on the 2D plane of the Z-Ra plane (Fig. 5). Moreover, there are countless surfaces around the axis of symmetry Z. This paper seems to contradict this theorem. However, as shown in Fig. 5, the convection, magnetic field, and current paths intersect without difficulty, and this arrangement seems to be logical. This theorem is discussed in terms of a single electromagnetic induction equation. Therefore, there is no viewpoint of mutual influence between distant electric circuits such as mutual inductance. On the other hand, in this paper, mutual inductance and Lenz's law are considered and discussed in a system of electromagnetic induction equations. Since the methods of discussion are different, we do not believe it is appropriate to apply this theorem to this paper.

Theorem 3. According to Cowling's theorem[3], it is impossible for an axially symmetric field to be self-maintained.

This paper contradicts this theorem. However, Cowling's theorem discusses whether poles arise in terms of a single time-independent electromagnetic induction equation. Additionally, the concept of magnetic field stability is different. Since the arguments are different, we believe it is not appropriate to apply this theorem to this paper for the same reasons described in the above theorem.

Theorem 4. A purely toroidal flow, that is, one with $u = \boldsymbol{\nabla} \times T_r$, cannot maintain a dynamo[13]. Note that this means that there is no radial motion, $u_r = 0$.

In this paper, toroidal convection is not discussed, so this theorem is not covered.

Similar yet Different Arguments

We discuss the relationship between this argument and similar claims. The following lists three examples.

1 Berechnung der mittleren Lorentz-Feldstärke $\overline{\mathfrak{b} \times \mathfrak{B}}$ für ein elektrisch leitendes Medium in turbulenter, durch Coriolis-Kräfte beeinflußter Bewegung[6].

2 Dynamos driven by poloidal flow exist[7].

3 Dynamo action in simple convective flows[8].

However, these papers discuss axisymmetric magnetic fields due to approximate convection averaging helicity or nonlinear convection. This differs from the argument of this paper in that it deals with truly simple poloidal convection.

In addition, this paper is characterized by the fact that it includes inductance, especially mutual inductance, and solves it as an eigenvalue problem. There are other discussions that address mutual inductance.

As an example, according to a book[14] published in a collection of several papers, the calculation of the α effect in the geomagnetic field is explained in relation to inductance using the electrical and mechanical mechanisms of disk generators as equivalent mechanisms. To our knowledge, inductance is used in all the different arguments from this paper.

Limitations and Expectations of this Thesis

In this paper, we do not perform numerical calculations on the behavior of convection. In particular, the stability of the magnetic field is an important issue, but since it is thought to depend on the behavior of convection, this paper lacks evidence in this regard. However, we believe that there are situations where it is useful because we were able to show the possibility of growing axisymmetric magnetic fields with novel concepts.

For instance, we mentioned earlier that we do not know whether the fourth term on the left side of Eq. (12) is powerful in convection in nature. In the case of convection, the power of this term is considered weak. We believe that it is possible to create arbitrary convection to some extent in an artificial plasma experimental facility rather than in the natural world, so the results of this research may be useful when aiming to strengthen the magnetic field. In this way, this paper is in the field of so-called magnetohydrodynamics, and we expect that it will be useful not only for astronomical bodies but also for research in plasma furnaces and sodium experimental facilities.

Conflicts of interest The author has no conflicts of interest to declare.

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