Part II: Formal Structure of the Hierarchical Information Propagation Model

Mathematical Foundations of Hierarchical Folding, Nonlinear Gauge Symmetry,

and Effective Geometric Deformation in Finite-Field Information Dynamics

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Abstract

This work develops the mathematical core of the Hierarchical Information Propagation Model (HIPM), independent of physical interpretation. Information states are defined on discrete hierarchical spaces over a finite field, and inter-level propagation is formalized by mixed linear–nonlinear operators. A nonlinear gauge symmetry associated with redundant internal representations is introduced. Folding operations and their fixed points are defined algebraically. We further derive the hierarchical Jacobian associated with inter-level maps and introduce an abstract effective metric structure induced by local deformation of these Jacobians. Time and gravitational modulation are formulated as meta-hierarchical deformations without assigning any physical interpretation. The formalism presented here provides the minimal structural backbone required for subsequent applications in higher parts of the framework.

Keywords

hierarchical information model, finite field, nonlinear gauge symmetry, folding operator, hierarchical Jacobian, effective metric, meta-hierarchical modulation.

1 Overview of the Hierarchical Information Propagation Model (HIPM)

The Hierarchical Information Propagation Model (HIPM) formalizes information flow across nested computational layers. Each layer represents a distinct mode of effective physical description: the classical layer, the quantum layer, and the folded (collapsed) layer. HIPM provides a unified framework to describe how information is generated, propagated, folded, and thermodynamically constrained.

The model consists of three primary components:

- **Classical Layer** ($\mathcal{H}_{\text{cl}}$): emergent deterministic variables and coarse-grained states.
- **Quantum Layer** ($\mathcal{H}_q$): superposition, entanglement, and reversible unitary evolution.
- **Folded Layer** ($\mathcal{H}_f$): information-reduced states resulting from layer – to – layer projection and non-linear folding dynamics.

HIPM asserts that information propagation between layers is not merely a projection but a thermodynamically constrained transformation. Transitions such as quantum-to-classical reduction and classical-to-quantum lifting are implemented by folding operators C_k and unfolding operators D_k.

These operators are defined over the effective map

$$\mathcal{H}_q \longleftrightarrow \mathcal{H}_{\text{cl}} \longrightarrow \mathcal{H}_f,$$

and may be inherently non-linear due to information compression and the loss of distinguishability under coarse-graining.

The goal of Part II is to define these operators precisely, illustrate their geometric structure, and introduce the concept of a local folding map, which encodes how information collapses in finite, bounded regions.

2 Three-Layer Structure and Folding Dynamics

2.1 Three-Layer Hilbert Structure

We formalize the three effective state spaces as follows:

$$\mathcal{H}_q : \text{Quantum layer (unitary, reversible)}, \tag{1}$$

$$\mathcal{H}_{\text{cl}} : \text{Classical-information layer}, \tag{2}$$

$$\mathcal{H}_f : \text{Folded layer (information-compressed)}. \tag{3}$$

A system state is represented as

$$|\Psi\rangle \in \mathcal{H}_q, \qquad X \in \mathcal{H}_{\text{cl}}, \qquad F \in \mathcal{H}_f.$$

2.1.1 Layer-to-Layer Operators

Information transitions are modeled with two families of operators:

- **Folding operators** C_k: quantum/classical states $\to$ folded states (irreversible, entropy-increasing).
- **Unfolding operators** D_k: folded states $\to$ classical/quantum states (partial reversible reconstruction).

Each operator is defined locally:

$$C_k : \mathcal{H}_q \cup \mathcal{H}_{\text{cl}} \longrightarrow \mathcal{H}_f,$$

$$D_k : \mathcal{H}_f \longrightarrow \mathcal{H}_{\text{cl}} \cup \mathcal{H}_q.$$

By construction,

$$D_k C_k \neq \mathbb{I},$$

which represents the fundamental information-loss condition consistent with Landauer-type thermodynamic constraints.

2.2 Local Folding Map

For a finite region Ω, HIPM defines a local folding map

$$\mathcal{F}_\Omega : \mathcal{H}_q(\Omega) \longrightarrow \mathcal{H}_f(\Omega),$$

which acts as a bounded, generally non-linear transformation encoding local collapse of information.

The total folding evolution along a timeline is given by the ordered composition

$$\mathcal{F}(t) = C_{k_t} \circ \cdots \circ C_{k_2} \circ C_{k_1},$$

illustrating the discrete nature of folding events.

This formalism naturally induces a hierarchical structure of layers:

$$\mathcal{H}_q \supset \mathcal{H}_{\text{cl}} \supset \mathcal{H}_f,$$

with each inclusion describing a loss of distinguishability and an increase in effective entropy.

3 Nonlinear Structure of Folding Operators

In HIPM, the essential distinction between layers arises from the nonlinearity introduced by folding transformations. While quantum dynamics in $\mathcal{H}_q$ is linear and unitary, and classical dynamics in $\mathcal{H}_{\text{cl}}$ can be modeled as stochastic yet linear in probability space, the transition into the folded layer $\mathcal{H}_f$ necessarily introduces nonlinear information compression.

This section formalizes the mathematical structure responsible for such nonlinearity.

3.1 Motivation: Why Folding Must Be Nonlinear

A folding operator

$$C_k : \mathcal{H}_q \cup \mathcal{H}_{\text{cl}} \;\longrightarrow\; \mathcal{H}_f$$

must satisfy two physical constraints:

1. **Information-reduction condition**

$$\dim(\mathcal{H}_f) < \dim(\mathcal{H}_q), \qquad \dim(\mathcal{H}_f) < \dim(\mathcal{H}_{\text{cl}}).$$

   This reflects coarse-graining and entropy increase.

2. **Non-reversibility of collapse** No operator D_k can satisfy

$$D_k C_k = \mathbb{I}$$

   on the entire domain.

These constraints imply that C_k cannot preserve convex combinations, and therefore cannot be linear. Formally, for generic states

$$C_k(\alpha A + \beta B) \neq \alpha C_k(A) + \beta C_k(B),$$

indicating that folding is intrinsically nonlinear.

3.2 Decomposition of the Folding Map

We decompose the folding operator C_k into two components:

$$C_k = \Pi_k \circ \Lambda_k,$$

where

- Λ_k: layer-to-layer preconditioning (linear map that extracts classical observables from quantum inputs),
- Π_k: nonlinear projection implementing coarse-graining.

3.2.1 Quantum Preconditioning

For quantum inputs,

$$\Lambda_k : \mathcal{H}_q \to \mathcal{H}_{\text{cl}},$$

we define

$$\Lambda_k(|\Psi\rangle) = \big(O_1(\Psi), O_2(\Psi), \ldots, O_m(\Psi)\big),$$

where $\{O_i\}$ are effective classical observables associated with a finite resolution scale. This preconditioning is linear due to the linearity of expectation values.

3.2.2 Nonlinear Projection

The projection

$$\Pi_k : \mathcal{H}_{\text{cl}} \to \mathcal{H}_f$$

is nonlinear and satisfies:

- many-to-one mapping (information compression),
- locality (acts on finite domains),
- entropy increase,
- metric contraction on distinguishability.

A central property is that for any two states X and Y,

$$d\big(\Pi_k(X), \Pi_k(Y)\big) \leq d(X, Y),$$

with strict inequality unless X and Y fall into distinct basins of the folding map.

3.3 Fixed Points and Stability

Folding maps generally possess stable fixed points:

$$F^* \in \mathcal{H}_f \quad \text{such that} \quad C_k(F^*) = F^*.$$

These points correspond to:

- locally maximally compressed descriptions,
- attractors under repeated folding,
- classical "collapsed" or "decided" states.

Under timeline evolution, a sequence

$$C_{k_t} \circ \cdots \circ C_{k_2} \circ C_{k_1}$$

generically converges toward a stable folded configuration.

This expresses mathematically how repeated information interactions drive the system toward irreversible collapse.

3.4 Partial Unfolding and Information Recovery

Unfolding operators

$$D_k : \mathcal{H}_f \to \mathcal{H}_{\text{cl}} \cup \mathcal{H}_q$$

are not true inverses but reconstruction mappings satisfying:

$$D_k C_k \approx \mathbb{I} \quad \text{on restricted domains.}$$

The reconstruction is approximate because:

- only coarse-grained observables survive the folding,
- fine quantum phases are unrecoverable,
- non-invertibility of Π_k prohibits exact reversal.

Thus, unfolding restores the *shape* of information flow but not its original microscopic detail.

3.5 Summary

This section clarified that:

- folding must be nonlinear due to information-loss requirements,
- the operator decomposes into linear preconditioning and nonlinear projection,
- repeated folding yields stable fixed points,
- unfolding operators provide only partial reconstruction.

The next section introduces the *geometric structure* of folding and visualizes information contraction within a multi-layer hierarchy.

4 Geometric Structure of Local Folding

The Hierarchical Information Propagation Model (HIPM) treats folding as a local geometric transformation acting on neighborhoods in the global state space. This section formalizes the geometric concepts underlying the folding map F_k and clarifies how layered structures $(\mathcal{H}_q, \mathcal{H}_f, \mathcal{H}_m)$ emerge from repeated local contractions.

4.1 Local Neighborhoods in the Global State Space

Let $\mathcal{H}$ denote the global information state space. For each position (or index) x_k we define its local neighborhood:

$$\mathcal{N}_k = \mathcal{N}(x_k;\, \rho),$$

where ρ is the locality radius, which may represent:

- spatial locality,
- causal adjacency,
- or information-theoretic coupling.

Each $\mathcal{N}_k$ is embedded in the global space by an inclusion map

$$\iota_k : \mathcal{N}_k \hookrightarrow \mathcal{H}.$$

A central assumption of HIPM is:

Folding acts locally, but propagates its effects globally.

Thus the folding operator F_k is defined on $\mathcal{N}_k$ and extended to the whole system via the induced propagation structure.

4.2 Definition of the Local Folding Map

The local folding map

$$F_k : \mathcal{N}_k \longrightarrow \mathcal{H}_f$$

acts in three conceptual steps:

1. extraction of the neighborhood block,
2. nonlinear folding within the block,
3. reinsertion into the global structure.

Formally:

$$F_k = r_k \circ \Pi_k \circ p_k,$$

where

- $p_k : \mathcal{H} \to \mathcal{N}_k$ (local projection onto the neighborhood),
- $\Pi_k : \mathcal{N}_k \to \mathcal{H}_f$ (nonlinear local fold),
- $r_k : \mathcal{H}_f \to \mathcal{H}$ (re-embedding into the global hierarchy).

This structure is consistent with the diagram depicted in Figure 4.

4.3 Geometric Contraction and Layer Mapping

Folding acts as a geometrically contracting map:

$$d\big(F_k(X), F_k(Y)\big) \le \gamma_k\, d(X,Y), \qquad 0 < \gamma_k < 1.$$

Repeated contraction generates an emergent hierarchy:

$$\mathcal{H} \;\longrightarrow\; \mathcal{H}_{L1} \;\longrightarrow\; \mathcal{H}_{L2} \;\longrightarrow\; \cdots \;\longrightarrow\; \mathcal{H}_f.$$

This process corresponds to the multi-layer diagram shown in Figure 1. Layers are defined recursively:

$$\mathcal{H}_{L(\ell+1)} = F_{k_\ell}\big(\mathcal{H}_{L(\ell)}\big).$$

Thus, $\mathcal{H}_{L0} = \mathcal{H}$ (global layer), $\mathcal{H}_{L1}$ is the first structured intermediate layer, and $\mathcal{H}_{L2}$ reflects compressed local neighborhoods.

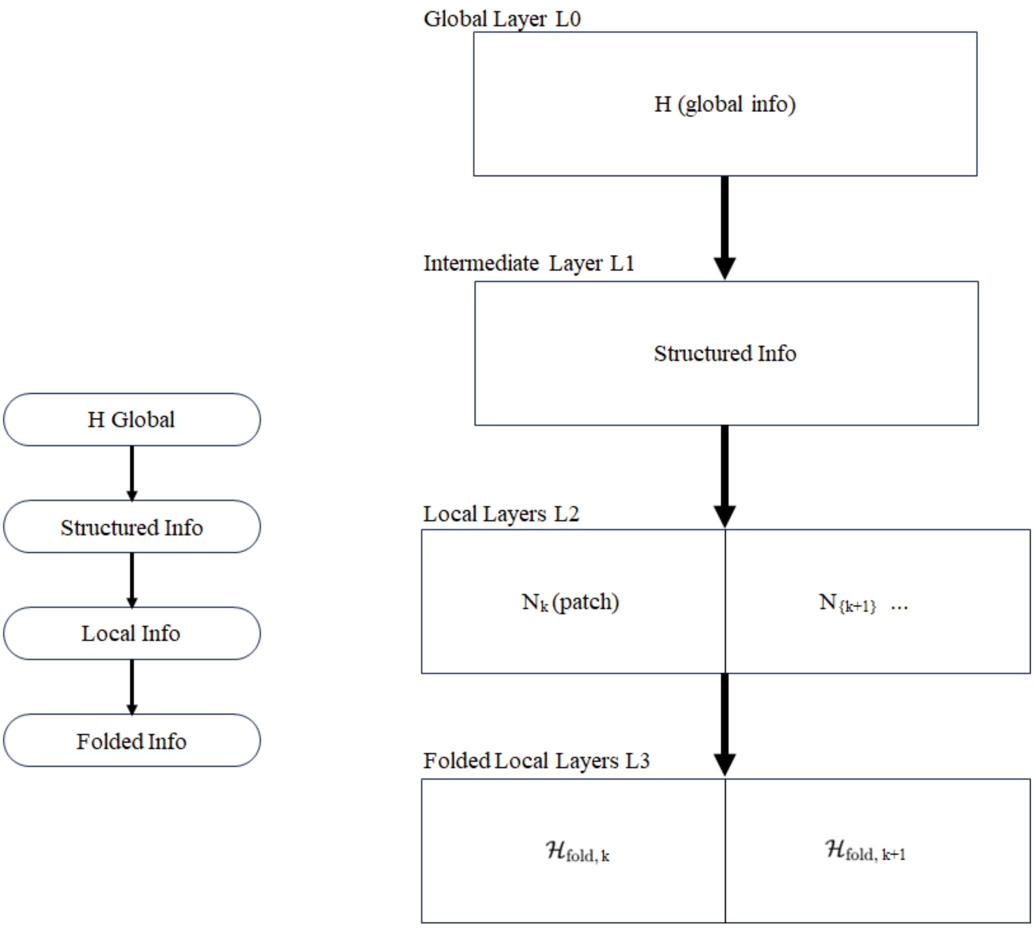

Figure 1: Recursive multi-layer structure showing global, intermediate, and local layers. This diagram corresponds to the hierarchical construction discussed in the text, where folding maps act across adjacent layers.

4.4 Relation to Quantum → Classical → Material Layers

In the HIPM physical interpretation, geometric hierarchy corresponds to three physically meaningful layers:

1. **Quantum Layer** $\mathcal{H}_q$ (high-dimensional, phase-sensitive structure),
2. **Folding Layer** $\mathcal{H}_f$ (nonlinear, entropy-increasing processing),
3. **Material Layer** $\mathcal{H}_m$ (stabilized, decohered classical outcomes).

These layers are related by the compositional map:

$$\mathcal{H}_q \xrightarrow{C_k} \mathcal{H}_f \xrightarrow{D_k} \mathcal{H}_m,$$

where C_k is the collapse (folding) operator and D_k is the divergence (unfolding/progression) mapping.

The timeline version of this structure corresponds to Figure 5.

4.5 Propagation Structure and Coarse Information Flow

Local folding at site k induces a propagation of reduced information toward adjacent neighborhoods. Let $\mathcal{P}_{k\to j}$ denote the propagation operator from site k to j.

Folding changes the global state by:

$$X \mapsto r_k(F_k(p_k(X))).$$

Neighboring regions are affected by:

$$X_j \mapsto \mathcal{P}_{k\to j}(F_k(p_k(X)))\,.$$

In particular, HIPM predicts:

local entropy increase spreads outward along causal adjacency.

This propagation structure is one of the signatures distinguishing HIPM from linear quantum evolution.

4.6 Layer Stabilization and Emergence of Material Configurations

As folding propagates across multiple layers, the system approaches stable, low-dimensional configurations:

$$\mathcal{H}_f \xrightarrow{D_k} \mathcal{H}_m,$$

where $\mathcal{H}_m$ corresponds to classical measurement results or material outcomes.

A stabilized material state satisfies:

$$M^* = D_k(F_k(\mathcal{N}_k)),$$

and becomes invariant under subsequent local contractions:

$$F_k(M^*) = M^*.$$

Thus material outcomes appear as fixed points of the geometric hierarchy.

4.7 Summary

This section established the geometric foundations of folding:

- Folding acts on local neighborhoods $\mathcal{N}_k$.
- The map decomposes into projection, nonlinear contraction, and reinsertion.
- Repeated local folding generates a multi-layer hierarchy.
- Quantum, folded, and material layers arise from this contraction process.
- Material outcomes correspond to stable fixed points in $\mathcal{H}_m$.

The next section analyzes the temporal evolution of these folding processes and formulates the dynamical equations governing information flow along the timeline.

5 Folding Dynamics and Timeline Evolution

This section formulates the temporal evolution of hierarchical folding in HIPM. Whereas Section **??** established the geometric structure of local folding, the present section introduces explicit dynamical equations describing how states in the quantum layer $\mathcal{H}_q$, folding layer $\mathcal{H}_f$, and material layer $\mathcal{H}_m$ evolve across discrete time steps $t_0 \to t_1 \to t_2 \cdots$.

We also show how the dynamics correspond directly to the folding timeline diagram presented in Fig. 5.

5.1 Temporal Indexing and Layered State Vectors

Let the full hierarchical state at time t_n be written as

$$X(t_n) = \big(H_q(t_n),\ H_f(t_n),\ H_m(t_n)\big).$$

Each component represents the (coarse-, intermediate-, and reduced-) information contained in the corresponding layer. The dynamics across time steps are generated by local folding maps F_k and divergence maps D_k, extended globally:

$$X(t_{n+1}) = \big(H_q(t_{n+1}),\ H_f(t_{n+1}),\ H_m(t_{n+1})\big).$$

5.2 Quantum-Layer Evolution: Pre-Folding Dynamics

Before collapsing into the folding layer, the quantum layer evolves under a reversible operator $U(t_n)$:

$$H_q(t_{n+1}^-) = U(t_n)\, H_q(t_n),$$

where t_{n+1}^- denotes the moment just before collapse/folding.

The operator U may be linear or nonlinear depending on whether HIPM is interpreted as a generalization of quantum mechanics or as a broader information-dynamical framework.

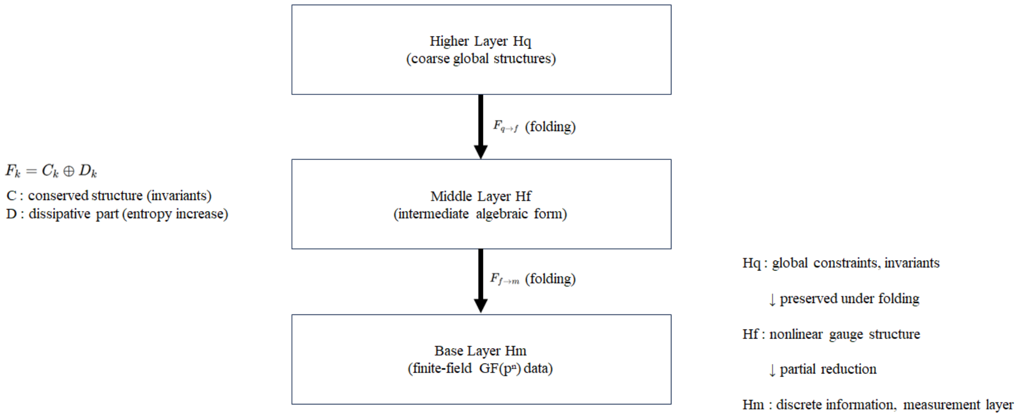

Figure 2: Overall relationship between quantum, folding, and material layers. Folding and divergence maps generate transitions between layers.

5.3 Folding Step: Collapse Operator C_k

At each site k, the collapse/folding operator C_k acts on the evolved pre-folded quantum state:

$$H_f(t_{n+1}) = C_k\big(H_q(t_{n+1}^-)\big).$$

This operator is the same as the local fold used in Section **??** and illustrated in Fig. 3. The essential physical property is contraction:

$$\|C_k(X) - C_k(Y)\| \le \gamma_k \|X - Y\|, \qquad 0 < \gamma_k < 1.$$

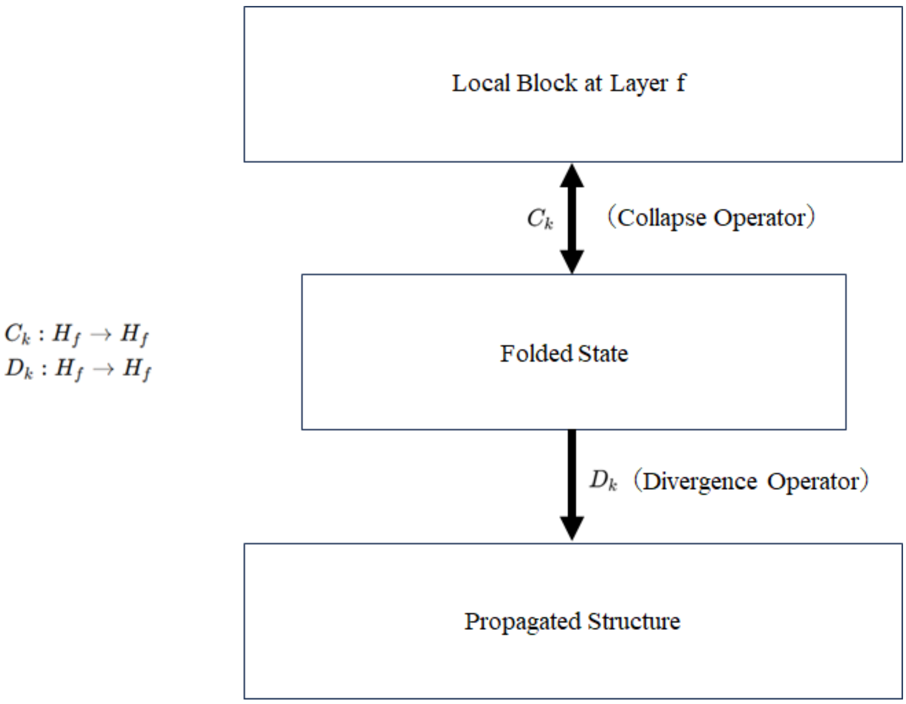

Figure 3: Collapse operator C_k and divergence operator D_k acting on the local block at layer f.

5.4 Local Map Propagation and Neighborhood Interaction

The local folding map F_k described in Section **??** induces changes in neighboring regions through the propagation operator $\mathcal{P}_{k\to j}$:

$$H_f^{(j)}(t_{n+1}) = \mathcal{P}_{k\to j}\big(F_k(\mathcal{N}_k)\big).$$

This structure matches the diagram shown in Fig. 4.

Propagation is typically dissipative:

$$S\big(H_f^{(j)}(t_{n+1})\big) \geq S\big(H_f^{(j)}(t_n)\big),$$

where $S(\cdot)$ is the entropy-like measure introduced in Section **??**.

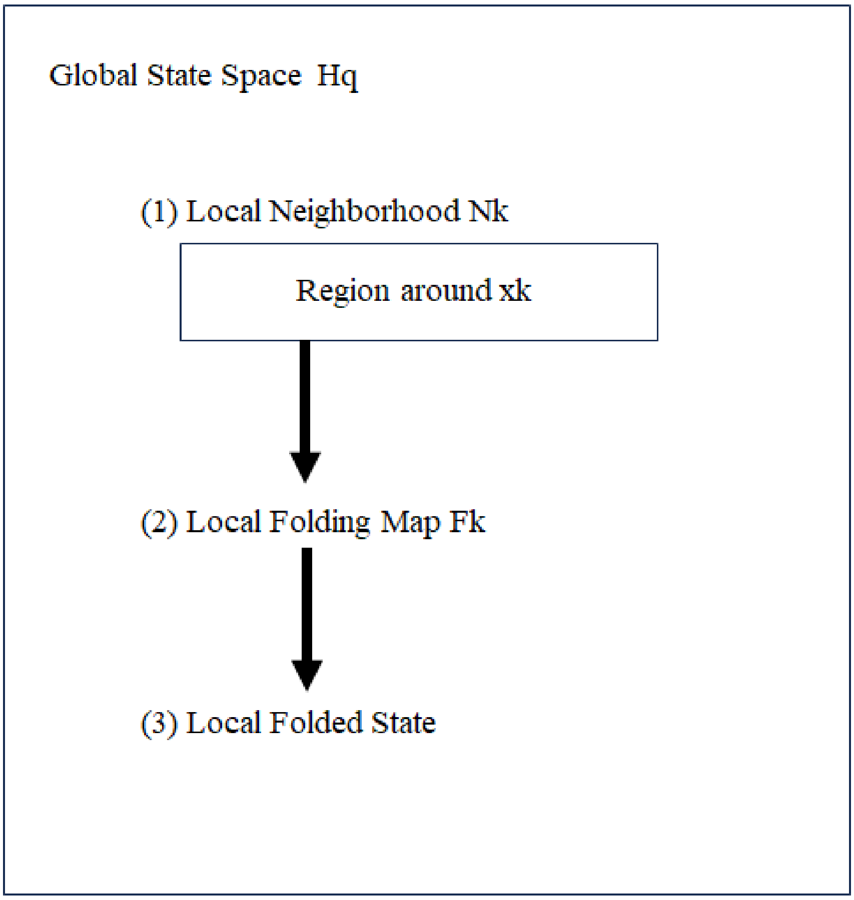

Figure 4: Local neighborhood extraction, local folding map F_k, and the resulting folded state.

5.5 Progression Toward Material Layer: Divergence Operator D_k

After the folding step, the system evolves further under the divergence map:

$$H_m(t_{n+1}) = D_k\big(H_f(t_{n+1})\big),$$

where D_k expands or stabilizes folded information into classical or material configurations.

D_k typically satisfies:

$$D_k(D_k(X)) = D_k(X),$$

meaning material outcomes are fixed points of continued divergence.

5.6 Hierarchy Stabilization Over Time

Repeated folding – divergence steps generate a temporal sequence:

$$H_q(t_0) \to H_f(t_1) \to H_m(t_2) \to H_m(t_3) \to \cdots$$

Over long timescales, the system approaches a stabilized material structure H_m^*:

$$H_m^* = D_k(F_k(\mathcal{N}_k)), \qquad F_k(H_m^*) = H_m^*.$$

5.7 Timeline Diagram and Layer Evolution

The above dynamics correspond directly to the layer-by-layer timeline shown in Fig. 5. Each horizontal arrow represents evolution within a layer, while each vertical arrow represents a C_k or D_k operation.

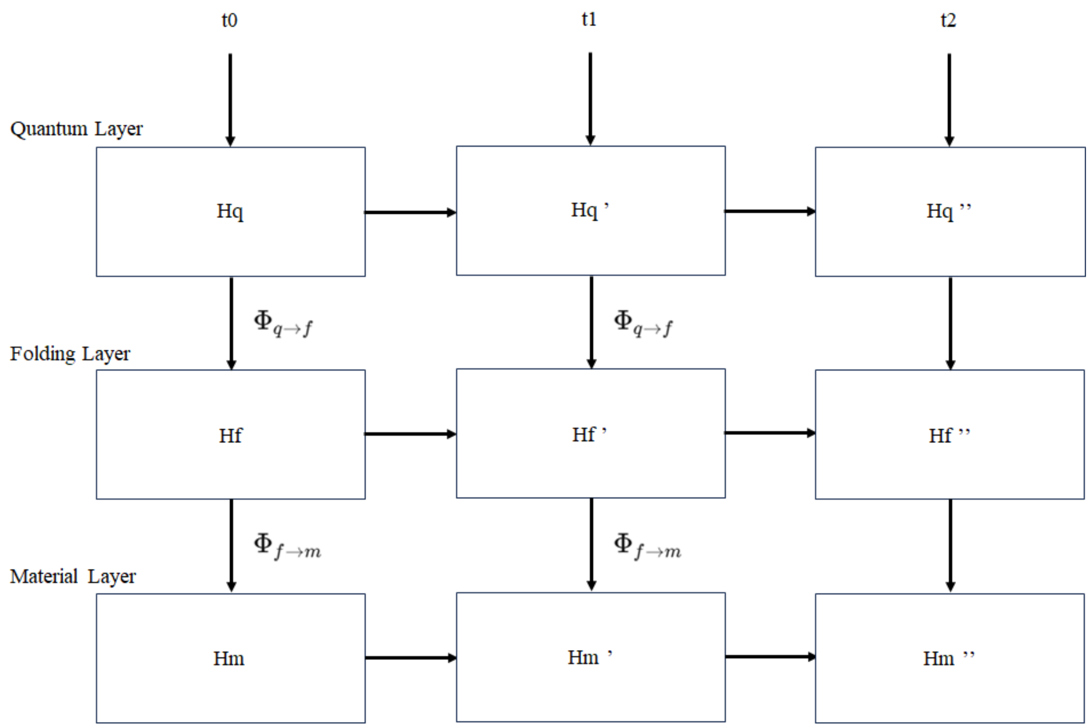

Figure 5: Timeline evolution of quantum, folding, and material layers. Vertical arrows indicate folding (C_k) and divergence (D_k).

5.8 Summary of Dynamical Structure

This section presented a complete dynamical formulation for HIPM folding:

- The quantum layer undergoes unitary or reversible evolution $U(t)$.
- At discrete times, local collapse/folding operators C_k map $\mathcal{H}_q$ into $\mathcal{H}_f$.
- Folding propagates via neighborhood interactions and increases entropy.
- Divergence operators D_k stabilize folded states into material configurations.

- Timeline evolution matches the multi-layer diagram of Part II.

This completes the core theoretical structure of Part II. Next, Part III can examine specific applications, simulation results, or finite-field realizations of the HIPM folding dynamics.

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